Assessing Fairness and Equity in Trajectory Based Operations

Isabel del Pozo y de Pozo\textsuperscript{1} and Dr. Miguel A. Vilaplana Ruiz\textsuperscript{2}

\textit{Boeing Research and Technology Europe, Madrid, Spain}

\textit{and}

Dr. Colin Goodchild\textsuperscript{3}

\textit{University of Glasgow, Glasgow, United Kingdom}

The focus of the work presented in this paper is set on defining a framework and the corresponding process to develop a fairness and an equity metric for a given cost model. This paper provides a definition for a just framework in ATM as well as the definition of the concept of fairness and equity in Trajectory Based Operations. Two metrics are proposed, one for evaluating fairness and one for evaluating equity, based on an example for a cost index based cost model.

\textbf{Nomenclature}

\begin{itemize}
  \item \(C\) = Cost Function
  \item \(C_{\text{fixed}}\) = Fixed Cost
  \item \(C_{\text{variable}}\) = Variable Cost
  \item \(C_T\) = Coefficient for time-related cost
  \item \(C_F\) = Coefficient for fuel-related cost
  \item \(e\) = generic variable
  \item \(F\) = Fuel consumption
  \item \(F_M\) = Modified fuel consumption
  \item \(F_P\) = Preferred fuel consumption
  \item \(F_{\text{REF}}\) = Reference fuel consumption
  \item \(F_{\text{SAT}}\) = Saturated fuel consumption
  \item \(\text{max}\) = Index to indicate the maximum value
  \item \(\text{min}\) = Index to indicate the minimum value
  \item \(P\) = Penalty Function
  \item \(P_{\text{SAT}}\) = Saturated Penalty Value
  \item \(T\) = Flight duration
  \item \(T_M\) = Modified flight duration
  \item \(T_P\) = Preferred flight duration
  \item \(T_{\text{REF}}\) = Reference flight duration
  \item \(T_{\text{SAT}}\) = Saturated flight duration
  \item \(E\) = Equity metric
  \item \(\Phi\) = Fairness metric
  \item \(\kappa\) = generic variable
  \item \(\tilde{\varphi}\) = Relative penalty function
\end{itemize}

\textsuperscript{1}Research Engineer, Advanced Trajectory Technologies, Isabel.delpozodepoza@boeing.com

\textsuperscript{2}Technical Manager, Advanced Trajectory Technologies, Miguel.vilaplana@boeing.com

\textsuperscript{3}Senior Lecturer, Air Traffic Management & Avionics, Department of Aerospace Engineering, c.goodchild@aero.gla.ac.uk
I. Introduction

A common understanding of how to measure the performance of the Air Traffic Management (ATM) system is key to understand and evaluate how new concepts and tools can improve the system. The number of ATM-related metrics is increasing but there is still a lack of standard methodology. The terminology used to describe these metrics often leads to ambiguity and misunderstanding, because different studies and research communities may have definitions differing from each other for the same term. Depending on the ATM stakeholder defining the metric, one can find different terminologies addressing the same metric or identical terminologies referring to different metrics.

Metrics can easily be misinterpreted due to the complexity of measuring activities in the airspace and the lack of standardized definition of metrics. In ATM, there is a need for a common understanding of ATM performance. Due to this lack of standardization, it seems impossible to compare the results and conclusions from different research studies.

A metric should represent a calculation guideline and according to this, a formula. A metric also has to define units in which the measurement is to be expressed.

In view of the ongoing ATM modernization initiatives in US and Europe (Single European Sky ATM Research (SESAR) and Next Generation (NextGen) respectively), there is a need for developing metrics that can assess the performance of an international information handling network for sharing data, of the new proposals for communication systems and protocols, as well as the automated Decision Support Tools (DST) required to implement the new Trajectory Based Operational (TBO) concepts.

The focus of this paper is on the fairness and equity metrics that need to be developed in order to be able to evaluate and compare the influence of those new automated DSTs on the performance of the ATM system.

For example, future Separation Assurance Systems (SAS) should be able to accommodate the user preferences and find an equilibrium among the different interest of the users. Thus, metrics assessing the fairness of those systems, or any system having to deal with accommodating user-preferences, will play a major role in the trajectory based operations of the future concepts proposed by SESAR and NextGen.

The remainder of this paper is structured as follows: first the concepts of fairness and equity in ATM are defined, the third chapter describes the development of the proposed fairness and equity metric, the fourth chapter analyses the metrics and the last chapter presents the conclusions.

II. Towards a Definition of the Concepts of Fairness and Equity in ATM

The ATM system has two key stakeholders: users and service providers. Both sides have different business interests, and both sides need to define a way to cooperate in the best interest of all, as both sides need each other. Those two sides are best represented by the Air Navigation Service Providers (ANSP) providing the air navigation service and the airlines requesting, among other, this air navigation service.

The ANSP providing services to the civil airspace users are represented throughout the world by an organization called Civil Air Navigation Services Organization (CANSO).

Similarly, most airlines are represented by an international industry trade group called International Air Transportation Association (IATA).

These organizations, CANSO and IATA, lead, serve and represent the interests of the main two groups that can be found in the ATM system.

In order to provide a just ATM system that covers the needs of all its stakeholders, the International Civil Aviation Organization (ICAO) was founded to supervise the entire ATM system. ICAO codifies “the principles and techniques of international air navigation and fosters the planning and development of international air transport to ensure safe and orderly growth.”

The main task of ICAO is to ensure a just ATM system by defining standards, procedures, functions and responsibilities for the ATM community to adopt. However, a clear definition of what a “just” ATM system is does not exist. Associated to the expression “just system”, people usually associate the expression of a “fair system”. So, what is a fair system and how can it be measured? These are key questions that have guided this work.

To guarantee the fairness of the ATM system is of extreme importance. To that aim, a common basis to compare results is required as well as transparency of the ATM related processes.

A. The Concept of Justice

The definition of justice or of something just is a central piece of human ethics, morality and philosophy. The concept of justice is language independent and a universal human idea that can be discussed from a philosophical point of view without any special attention to the semantic of individual languages.
The concept and its foundations have been considered many times throughout history, from the Greeks philosophers, starting with Aristotle, until today, and from a variety of perspectives.

In his book “A Theory of Justice” John Rawls states that, in order to ensure justice, one needs to start at the so called “original position”6 or “veil of ignorance”7. In this original position, no one is aware of her or his own incentives, and so has no tendency toward selfish behavior.

The original position is the premise and a requirement before agreeing upon standards and laws that will define the just framework for society. According to Rawls’ philosophy, to ensure justice, the principles of justice have to be selected “by all whom they apply under conditions preventing them from tailoring the principles to their own advantage.”8

In the ATM system, ICAO, which is assumed to be impartial, defines the standards of what is right and wrong, what are the responsibilities of the different parties and the duties and obligations they may have among each other. These standards have to be agreed among all member of the ATM system and represent the legal basis and common measure to decide all controversies between the ATM stakeholders. Thus, ICAO is responsible for the definition of a just framework for the ATM system.

B. The Concept of Fairness

The words justice and fairness are often used interchangeably because their meanings and usages are so closely linked despite their distinct connotations9. Conceptual differences exist between justice and fairness even though there is not always a distinct word for the two concepts in all languages. Fairness or something fair does not have a one to one translation from English to other languages5. But the concept of fairness and the word “fair” evolve as the result of certain situations that become more common during the Industrial Revolution. Those situations involve tradeoffs in welfare between individual. Thus, people start to develop a common thinking of the limits to how much one person is allowed to cost others in order to benefit her or himself5,10.

Fairness implies achieving a balance of conflicting interests and represents a potential tension between what someone wants to do and what can be bad for another within a just framework. It is inherently relational, as one’s actions affect someone else5.

Fair is related in meaning to equity, just, and acceptable limits. Unconsciously each person has a sense for what is acceptable and what not, thus a sense of what is fair and not fair.

C. The Concept of Equity

Equity is a special case of fairness. Equitable implies equal treatment of all concerned11.

In order to indentify the differences between just, equitable and fair, the following example is proposed after reference 8.

Let us consider two hungry friends, each wanting to buy a slice of pizza, but with only one slice left. In this case, it would be just for them to share the slice of pizza. However, there are two ways of sharing the slice of pizza, and both of them within the just framework of “sharing the slice of pizza”. Depending on their subjective sensation of what is acceptable, the friends can divide the slice in an equitable or fair manner:

- Equitable would be to divide the single slice of pizza in two identical portions, one for each of the two friends.
- Fair would be to take into account how hungry each of the two friends is and to divide the slice proportionally to each person’s hunger.

The latter case relies on the honesty of the two friends. It assumes both friends have agreed a common method to measure and express hunger and both would not lie when they express their personal hunger. This manner of sharing is fair if these assumptions do not fail. If it can be proven that one of both friends was not honest and this gave him an advantage in order to get a bigger part of the slice, then the fairness of this sharing method is undermined.

Only if both friends have the same hunger, then the fairest way of sharing the slice of pizza is also equitable.

D. Conclusions

To sum up, the definitions use in this paper for justice and just, fairness and fair, and finally equity and equitable are as follows:

- Justice or something just is being or acting in conformity with what is morally upright by following standards of what is right assuming that those standards were defined by those whom they apply to under conditions preventing them from tailoring the principles to their own advantage.
- Fairness or something fair is to achieve a balance between conflicting interests by means of a just procedure that takes into account the acceptance levels of a society and the satisfaction of the individuals. It is inherently relational.
- Equity or something equitable is a special case of fairness and suggest equal treatment of all concerned.

Regarding the ATM system, justice at an international level is provided and taken care off by ICAO. As long as the ANSPs and the airline act accordingly to the standards defined by ICAO, they are being and acting justly. But it always has to be assumed that these standards were agreed on under a common consensus of all ATM stakeholders (ANSP, Airlines, Airport, Government, etc).

Considering that the ATM system has two key stakeholders: users and service providers, fair and equitable is applicable when comparing how the service was provided.

Those two key stakeholders are represented by the ANSPs, providing the air navigation service, and the airlines, making use of that service.

### III. Developing the Metrics

When issuing a flight for the first time, the Airline Operation Centre (AOC) has to consider several factors for defining a flight path that suits the business strategy of the airline. Thus, the importance relies on all factors that affect the costs the airline has to consider for a given flight, factors as “fuel and personnel costs, alternate airports, route charges, traffic constraints, day of the year, infrastructure availability and weather including warning for significant meteorological information.”

The cost functions used by AOCs are complicated and airline dependant. Those cost functions are not public, at least not in detail. Thus, a general cost function is described as it can be found in the literature (see reference 13). By means of this cost function, the importance of the cost definition when deciding the trajectory of a specific flight can be understood. The role the cost function plays is relevant nowadays as well as for the future TBO scenarios proposed by SESAR and NextGen.

The cost function, also in its general expression, has flight specific coefficients that are defined by each airline according to its strategy. In order to explain the required tradeoff between these flight specific coefficients, the cost index is introduced.

Based on the relevance of the costs when defining a flight and its preferred trajectory according to the airline’s strategy, a simplified cost model is proposed. This model leads to the definition of the penalty concept and consequently to the penalty function, which represent the penalty level incurred when deviating from the airline’s defined cost for a specific flight. The penalty concept is key for defining the metric of fairness and the metric of equity, which is the aim of this paper.

The simplified cost model as well as the suggested penalty function should help as an example to illustrate the proposed metrics for fairness and equity in TBO and to obtain meaningful results that substantiated those metrics.

The fairness and equity metrics are independent of the form of the cost function or the penalty function but have to be compliant with the detailed assumptions and conditions stated in this chapter. Both metrics capture the definition of fairness and equity in ATM as defined in section II. As a brief recalling, fairness implies to achieve a balance between conflicting interests by taking into account the acceptance levels and satisfaction of individuals and is inherently relational. Equity is a special case of fairness and suggests equal treatment of all concerned. Equity is also inherently relational.

#### A. The Proposed Cost Model and its Assumptions

The cost model used for the analysis is based on following assumptions:

1) the additional cost incurred is calculated by the deviation from the airline’s preferred variable cost. This is reflected in the difference between the modified and the preferred values for flight duration and fuel consumption. Thus, as this difference increases the cost incurred also increases.

2) the airline does not incur in extra costs when the flight duration is shorter than the preferred duration or the fuel consumption is less than the preferred value or both. This assumption has been made intentionally to simplify the model.

3) the additional cost incurred is related to the values of modified flight duration and fuel consumption, $T_M$ and $F_M$ respectively. Each flight according to the airline’s strategy has a maximum acceptable incurred cost which is correlated to specific values for the increase in flight duration and fuel consumption. The maximum acceptable cost is defined by a pair of reference values of flight duration and fuel consumption, $T_{REF}$ and $F_{REF}$ respectively. The acceptable delay $\Delta T_{REF}$ is defined as the difference between the so defined “reference” flight duration $T_{REF}$ minus the preferred flight duration $T_{P}$. Similarly, the tolerable increase in
fuel consumption $\Delta F_{\text{REF}}$ is defined as the difference between the so defined “reference” fuel consumption $F_{\text{REF}}$ and the preferred fuel consumption $F_{\text{P}}$.

$$\Delta T_{\text{REF}} = T_{\text{REF}} - T_{\text{P}}$$
$$\Delta F_{\text{REF}} = F_{\text{REF}} - F_{\text{P}}$$  \hspace{1cm} (1)

The maximum tolerable increases in flight duration and fuel consumption ($\Delta T_{\text{REF}}$ and $\Delta F_{\text{REF}}$) represent the acceptance levels of each airline for each specific flight. These acceptance levels are defined by the airline through a pair of reference values ($T_{\text{REF}}$ and $F_{\text{REF}}$). Because fairness has to take into account the acceptance levels and satisfaction of individuals, the pair of reference values determined by the airline is relevant to the metric.

The reference values $T_{\text{REF}}$ and $F_{\text{REF}}$ do not constrain $T_{\text{M}}$ and $F_{\text{M}}$, which can trespass them, but they represent a pair of maximum acceptable levels as defined by the airline.

In an ideal situation, the airline provides this pair of maximum tolerable values $T_{\text{REF}}$ and $F_{\text{REF}}$ according to its strategy, the defined flight costs and their preferred trajectory. If this data is not available, then reference data from SESAR and NextGen may be used.

4) the additional cost incurred cannot increase indefinitely. The maximum additional cost is directly related to the feasible performance of the aircraft.

5) each additional cost represents a cost penalty to the airline’s strategy for a given flight. Different as for the additional cost incurred, the cost penalty has a maximum value which does not depend on the aircraft performance but on the value combination of $T_{\text{M}}$ and $F_{\text{M}}$. Having always in mind the definition of fairness in ATM, as the aim is to define a fairness metric, the cost penalty is maximal when the tolerable incurred cost defined by the airline through the pair $T_{\text{REF}}$ and $F_{\text{REF}}$ is reached or trespassed. Thus, $T_{\text{REF}}$ and $F_{\text{REF}}$ are an instrument at the airline’s disposal for expressing the maximum cost penalty it can tolerate according to its strategy.

6) the additional cost incurred, due to modification to the airline’s preferred trajectory, can keep on increasing until the feasible maximum. The maximum cost penalty is defined by the maximum acceptable additional cost provided by the airline through the values $T_{\text{REF}}$ and $F_{\text{REF}}$. Once that maximum acceptable additional cost has been reached, the cost penalty is maximal. For any further cost increments beyond the maximum acceptable additional cost, the cost penalty stays constant at its maximum value.

Thus, the maximum cost penalty can be reached for different combinations of values of $T_{\text{M}}$ and $F_{\text{M}}$ resulting from the proposed trajectory modifications that equal or trespass one of the acceptance levels $T_{\text{REF}}$ or $F_{\text{REF}}$.

B. The Cost Function in a Trajectory Based Operation Environment

In a trajectory based operation environment, trajectory-related information will become the main piece of information being shared. This trajectory-related information has to contain an accurate description of how a specific trajectory is intended to be flown (regarding user preferences like how a specific airline prefers to fly descent profiles) and how an aircraft is intended to be operated to follow that trajectory within a timeframe (detailed speed, lateral and altitude profiles as well as aircraft configuration e.g. flaps).

Within the TBO concept, the airline will have to provide to the ANSP its preferred trajectory or trajectory segment for a given flight prior to that flight flying the assigned trajectory or trajectory segment.

The preferred trajectory describes implicitly the preferred costs by defining the preferred flight duration and fuel consumption.

The preferred cost can only be achieved if the preferred trajectory is flown and this is only possible in an ideal situation. Usually, the trajectory has to be modified due to unpredictable weather conditions, unexpected traffic congestions or due to any other modifications required by the ANSP in order to ensure the safety of the operations.

The total flight cost is reflected in the cost function. Each cost function has fixed costs and variable costs depending on the airline’s strategy.

$$C = C_{\text{fixed}} + C_{\text{variable}}$$ \hspace{1cm} (2)
The fixed cost $C_{\text{fixed}}$ comprehends for example the insurance costs, the personal equipment costs, the costs for uniforms, and crew trainings costs and is by definition independent of the way the trajectory is flown and of the flight duration.

The variable cost $C_{\text{variable}}$ is, in its simplified version, a function of the flight time related cost and of the cost associated to the fuel consumption for a given flight:

$$C_{\text{variable}} = C_T \cdot \Delta T + C_F \cdot \Delta F$$

where $C_T$ and $C_F$ are flight specific coefficients integrated in the cost function for each flight according to the airline’s strategy. $C_T$ defines the variable time-related cost of a flight and $C_F$ defines the variable fuel-related cost of fuel.

The aim of the AOC is to minimize the total flight costs. In order to do so, the variable cost needs to be minimized.

The variable cost is split into time related and fuel related cost. Because the airline’s aim is to minimize the variable cost in order to optimize the cost function, that optimization has two objectives:

- to minimize the flight duration
- to minimize the fuel consumption.

The cost index shows how the airline weighs these two objectives, namely the relative importance of reducing the time related cost and the fuel related cost against each other for a specific flight.

The ratio between the flight specific $C_T$ for the time-related cost and the flight specific $C_F$ for the cost of fuel defines the cost index CI:

$$CI = \frac{C_T}{C_F}$$

During flight execution, the Flight Management Systems (FMS) calculates the most efficient trajectory, namely the one that minimizes the cost defined by the CI input by the pilot or the AOC.

The preferred user-defined cost function is the one that minimizes the total flight cost. The trajectory that results when considering all the factors in order to minimize the flight cost, and obtain the preferred user-defined cost function, is the User Preferred Trajectory (UPT).

The deviation of the preferred trajectory and consequently the increase in the incurred cost is reflected in two main variables, relevant for the variable cost, namely the difference in the flight duration, $\Delta T$, and the difference in the fuel consumption, $\Delta F$. These two differences are determined by comparing the preferred values for flight duration $T_P$ and fuel consumption $F_P$ against the values resulting from the modified trajectory for flight duration $T_M$ and fuel consumption $F_M$.

$$\Delta T = T_M - T_P$$
$$\Delta F = F_M - F_P$$

C. The Penalty Function

There is a need to define a function able to evaluate the additional cost incurred when a flight is deviated from the preferred trajectory and the cost penalty associated to it. That function has to comply with the six model assumptions described before and serve as the basis to evaluate the fairness and equity of a process, system or method handling and modifying trajectory related information.

Let that function be called the penalty function, which is a function of the resulting values from the modified trajectory for flight duration $T_M$ and fuel consumption $F_M$ and the preferred values for flight duration $T_P$ and fuel consumption $F_P$.

$$P = f(T_M, F_M, T_P, F_P)$$

The modified values $T_M$ and $F_M$ as well as the preferred values $T_P$ and $F_P$ have to comply with following properties:
A) $T_P$ is a fixed value for the preferred flight duration which is defined by the airline and can be either declared by the airline for a given flight or it can be deduced from the preferred trajectory for that flight, which is declared by the airline. $T_P$ is always a positive value, $T_P \in \mathbb{R}^+$

B) $F_P$ is a fixed value for the preferred fuel consumption which is defined by the airline and can be either declared by the airline for a given flight or it can be deduced from the preferred trajectory for that flight, which is declared by the airline. $F_P$ is always a positive value, $F_P \in \mathbb{R}^+$

C) $T_M$ is a variable value for the flight duration of a given flight which depends on the modification made to the preferred trajectory. The exact value can be obtained from the data resulting when the flight has flown the trajectory, or from the predictions made to, for example, analyze the what-if scenarios for different trajectory modifications. $T_M$ is always a positive value, $T_M \in \mathbb{R}^+$

D) $F_M$ is a variable value for the fuel consumption of a given flight which depends on the modification made to the preferred trajectory. The exact value can be obtained from the data resulting when the flight has flown the trajectory, or from the predictions made to, for example, analyze the what-if scenarios for different trajectory modifications. $F_M$ is always a positive value, $F_M \in \mathbb{R}^+$

E) $T_{REF}$, according to the third assumption of the presented model, defines (together with $F_{REF}$) the maximum acceptable incurred cost for a given flight. The values for $T_{REF}$ are always positive and equal or greater than the values for $T_P$: $T_{REF} \geq T_P$

F) $F_{REF}$, according to the third assumption of the presented model, defines (together with $T_{REF}$) the maximum acceptable incurred cost for a given flight. The values for $F_{REF}$ are always positive and equal or greater than the values for $F_P$: $F_{REF} \geq F_P$

The required penalty function has to obey following constraints in order to fit into the defined framework described by the proposed model and the characteristics of $T_M$, $F_M$, $T_P$ and $F_P$:

i) according to the second assumption of the proposed model, no cost penalty is incurred when $T_M$ is equal or less than $T_P$ and $F_M$ is equal or less than $F_P$. In such cases, the penalty function is equal zero.

$$P(T_M, F_M, T_P, F_P) = 0 \mid T_M \leq T_P, F_M \leq F_P$$ (7)

ii) according to the first assumption of the proposed model, the penalty function adopts a strictly positive value, when $T_M$ is greater than $T_P$ and for any $F_M$

$$P(T_M, F_M, T_P, F_P) > 0 \mid T_M > T_P$$ (8)

iii) according to the first assumption of the proposed model, the penalty function adopts a strictly positive value, when $F_M$ is greater than $F_P$ and for any $T_M$

$$P(T_M, F_M, T_P, F_P) > 0 \mid F_M > F_P$$ (9)

iv) according to the fifth assumption of the proposed model, the cost penalty has a maximum value, thus the penalty function is saturated when that maximum value is reached. The maximum value is referred to as $P_{SAT}$.

v) the penalty function reaches $P_{SAT}$, for example, when $T_M$ is equal $T_{REF}$ and when $F_M$ is equal $F_{REF}$. The maximum acceptable cost, as determined by the airline, corresponds to the maximum acceptable delay and maximum tolerable increase in fuel consumption reflected in the reference values. $T_{REF}$ and $F_{REF}$ are in turn used to define the value of the maximum cost penalty $P_{SAT}$:

$$P_{SAT} = P(T_M = T_{REF}, F_M = F_{REF}, T_P, F_P)$$ (10)
P_{SAT} could be equal zero for the case where T_{REF}=T_p and F_{REF}=F_p. In order to avoid that case, either T_{REF} or F_{REF} (or both) must be strictly greater than T_p and F_p, respectively. This means, that at least one of the two following inequalities must be true:

\begin{align}
T_{REF} &> T_p \\
F_{REF} &> F_p
\end{align}

(11)

vi) according to the sixth assumption of the proposed model, the penalty function is also saturated for certain combinations of values for T_M and F_M. There are two special cases, where P_{SAT} is reached even if one of the values of T_M or F_M is maintained at its preferred level (T_p or F_p, respectively).

For the case where F_M equals F_p, P_{SAT} is reached for a value of T_M, denoted as T_{SAT}, which is implicitly defined as:

\begin{align}
P(T_M = T_{SAT}, F_M = F_p, T_p, F_p) = P_{SAT}
\end{align}

(12)

Corollary to v) and vi):

\begin{align}
T_{SAT} \geq T_{REF}
\end{align}

(13)

vii) For the case where T_M equals T_p, P_{SAT} is reached for a value F_M, denoted as F_{SAT}, which is implicitly defined as:

\begin{align}
P(T_M = T_p, F_M = F_{SAT}, T_p, F_p) = P_{SAT}
\end{align}

(14)

Corollary to v) and vii):

\begin{align}
F_{SAT} \geq F_{REF}
\end{align}

(15)

viii) as requested by the first assumption of the proposed model, any increment in the values of T_M or F_M or both results in an increment of the penalty value because any increment of T_M and F_M with respect to T_p and F_p represents an additional cost incurred in flying the modified trajectory. According to the sixth assumption, the maximum value for the penalty function is P_{SAT}. Thus, the penalty function is a monotonically increasing function, which is comprehended between the values zero and P_{SAT}.

\begin{align}
P \in [0, ..., P_{SAT}]
\end{align}

(16)

Any function P can be used as penalty function as long as the set of variable properties described in A) to F) and the set of mathematical requirements in i) to viii) are respected.

Recalling the definition of the concept of justice, in order to provide a just framework for the analysis, the airlines and the ANSPs have to agree on how to express the penalty function.

D. The Proposed Penalty Function

For the further analysis, a specific penalty function is proposed. The penalty function can be defined as the function depicting the relationship between the additional time related cost and the additional fuel related cost incurred with reference to the preferred time related and fuel related cost.

The proposed penalty function is defined as follows:

\begin{align}
P = \sqrt{C_T^2 (T_M - T_p)^2 + C_F^2 (F_M - F_p)^2}
\end{align}

(17)
where \( C_T \) and \( C_F \) are flight specific coefficients with fixed values defined by the airline for each flight according to the airline’s strategy.

Following table summarizes the penalty function for the different intervals of \( T_M \) and \( F_M \):

<table>
<thead>
<tr>
<th>( P(T_M, F_M, T_p, F_p) )</th>
<th>( F_M )</th>
<th>( F_M \leq F_p )</th>
<th>( F_p &lt; F_M &lt; F_{SAT} )</th>
<th>( F_M \geq F_{SAT} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_M \leq T_p )</td>
<td>0</td>
<td></td>
<td>( C_F \sqrt{(F_M - F_p)^2} )</td>
<td>( P_{SAT} )</td>
</tr>
<tr>
<td>( T_p &lt; T_M &lt; T_{SAT} )</td>
<td>( C_T \sqrt{(T_M - T_p)^2} )</td>
<td>( \min \left{ \frac{C_T^2(T_M - T_p)^2 + C_F^2(F_M - F_p)^2}{P_{SAT}} \right} )</td>
<td>( P_{SAT} )</td>
<td></td>
</tr>
<tr>
<td>( T_M \leq T_{SAT} )</td>
<td>( P_{SAT} )</td>
<td>( P_{SAT} )</td>
<td>( P_{SAT} )</td>
<td></td>
</tr>
</tbody>
</table>

Following figures show a graphical representation of the proposed penalty function in 3D and 2D.

Figure 1. Penalty Value Limitations

Figure 2. Proposed Penalty Function in 2D

Figure 3. Proposed Penalty Function in 3D
The Iso-Penalty Curves are comparable to indifference curves in microeconomic theory. Any point of the same Iso-Penalty Curve represents a pair of \( T_M \) and \( F_M \) values for which the cost penalty reaches the same value. Thus, the airline has no preference for one point or another on the same curve.

As an example, imagine two airlines, Airline A and Airline B, with different business strategies for the same aircraft type covering the same route between the same two city pair, take for example from Madrid to Marseille.

Airline A has built on its image on punctuality and gain the reliance of its clients. In order to maintain its status, it is extremely important for Airline A to arrive on-time at its destination. When optimizing the variable cost, airline A weighs the time related cost as more important than the fuel related cost.

Imagine now Airline B whose image is based on selling cheap flight tickets. For this airline, the importance relies on maintaining its capacity of selling cheap flight tickets by constraining the total operational trip cost. When optimizing the variable cost, airline B weights the fuel related cost as more important than the time related cost. Thus, the cost index of airline A has a higher value than the cost index of airline B.

Let us assume that airline A has a \( CI=0.8 \) and airline B has a \( CI=0.4 \), and both have the same preferred flight duration \( T_p \) and fuel consumption \( F_p \). According to the defined cost index, airline A will reach the saturation value of the penalty function \( P_{\text{SAT}} \) for a smaller increase of the flight duration than airline B, but tolerate a greater increase of the fuel consumption before reaching \( P_{\text{SAT}} \). For airline B, it is the other way round. Airline B will reach saturation of the penalty function for a smaller increase in the fuel consumption, but it tolerates a greater increase in the flight duration than airline A before reaching \( P_{\text{SAT}} \).

Taking into account the weight the airline gives to the time related and fuel related cost helps to maintain the incurred cost closer to the preferred cost. That is one important reason for including the cost index as part of the function describing the cost penalty.

The flight is executed in a specific period of time for which the fuel price is assumed to be constant. Taken into account the definition for the cost index, then the penalty function can be expressed as:

\[
P = \sqrt{C_T^2 (T_M - T_p)^2} + C_F^2 (F_M - F_p)^2 = C_F \sqrt{CI^2 (T_M - T_p)^2 + (F_M - F_p)^2}
\]

with \( CI = \frac{C_T}{C_F} \) and assuming \( C_F=\text{const} \), then \( C_T = CI \cdot C_F \).

![Figure 4. Iso-Penalty Curves for airlines with different Cost Index operating the same aircraft on the same flight with same \( T_p \) and \( F_p \)](image)

E. The Relative Penalty Function

The objective followed in this paper is to compare the cost incurred when different trajectories, whether as a whole or a segment, are modified in the ATM system and define metrics that are able to evaluate whether those modifications took into account the user preferences for establishing the equity and fairness of the process, system or method deciding those modifications.
The defined penalty function reflects the extra cost incurred by one flight when the preferred cost determined by the airline’s strategy cannot be maintained. As detailed before, each flight has different values for the maximum penalty cost $P_{SAT}$, since it depends on $C_l$, $C_f$, $T_{REF}$, $T_{P}$, $F_{REF}$, and $F_{P}$. By comparing the different penalty values, it is difficult to draw any conclusions.

A common context needs to be provided to compare the values resulting from each penalty function for each flight. In such cases, using dimensionless variables paves the way to the required comparisons.

For that aim, a dimensionless penalty functions is defined, which is referred to as the relative penalty function:

$$\phi = \frac{P}{P_{\text{max}} + \kappa} = \frac{P}{P_{SAT} + \kappa}$$

(19)

where $\kappa$ is a strictly positive value much smaller than $P_{SAT}$.

$$0 < \kappa << P_{SAT}$$

(20)

The relative penalty function shows the percentage of the cost penalty that was incurred to a single flight compared to the maximum cost penalty value determined by the airline according to its strategy. This enables to compare different relative values for the incurred penalty among different flights.

The dimensionless penalty function has the following properties:

a) the dimensionless penalty function is maximal when the penalty function is also maximal:

$$\phi_{\text{max}} = \frac{P_{SAT}}{P_{SAT} + \kappa}$$

(21)

b) the dimensionless penalty function is minimal when the penalty function is also minimal:

$$\phi_{\text{min}} = \frac{0}{P_{SAT} + \kappa} = 0$$

(22)

The relative penalty function is a monotonically increasing function, which is comprehended between the values zero and one:

$$\phi \in [0,1]$$

(23)

F. The Fairness Metric

The fairness metric proposed in this work combines the geometric and the arithmetic mean. This way fairness metric evaluates whether the satisfaction has been distributed in a fair manner but it also penalizes the dispersion in the distribution. The proposed fairness metric is expressed as follows:

$$\Phi = \left( \prod_{i=1}^{n} \left(1 - \phi_i\right) \right)^{\frac{1}{n}} \cdot \frac{1}{n}$$

(24)

Recalling the definition of fairness described in chapter 2, the fairness metric has to comply with three key statements:

1) achieve a balance of conflicting interest by means of a just procedure
2) take into account the acceptance levels of a society and satisfaction of individuals
3) is inherently relational
In order to provide a just procedure to measure fairness, it has to be agreed by all whom it concerns to measure fairness according to the assumptions and constraints presented in this document. All involve have to define without self-regard a penalty function that complies with those assumptions and constraints. Once this just framework is provided, the proposed fairness metric can be used as it covers all the key statements of the definition of fairness.

The penalty function, on which this metric is based, reflects the balance of the two conflicting interests, namely to balance the time-related versus the fuel-related cost. This balance is achieved by incorporating in the penalty function the airline’s cost index, which weighs those two conflicting interests according to the airline’s strategy for a given flight. Thus, the first statement of the fairness definition is satisfied by this metric.

The relative penalty function describes the cost penalty incurred relative to the maximum cost penalty $P_{SAT}$, which is defined by the reference values $T_{REF}$ and $F_{REF}$. These reference values are an instrument at the airline’s disposal for expressing the maximum cost penalty it can tolerate according to its strategy for a given flight. Thus, the metric considers the acceptance levels defined by the airline.

The proposed fairness metric describes how far the additional incurred cost is from the maximal penalty cost, namely reflecting the satisfaction of the user, by including the formula \( (1 - \varphi) \). According to this, the second statement of the fairness definition is fulfilled.

Per definition, fairness is inherently relational. Thus, a fairness statement about how the trajectory of one single flight has been modified does not add a meaningful result. The aim is to measure the fairness of a system, method or process modifying different trajectories and trying to accommodate the user’s (here the airline) preferences regarding flight cost.

The proposed fairness metric has been defined to penalize the dispersion of the cost penalties. Hereby, the third statement of the fairness definition is contemplated.

The fairness metric relies on the honesty of the airlines when providing their reference values $T_{REF}$ and $F_{REF}$, which lead to $P_{SAT}$, and their cost index. If the airlines are untruthful when providing the data, then the correct measurement of the fairness cannot be guaranteed.

The proposed fairness metric is maximal when the relative penalty is the same in all cases. In that case the fairness is equal 1.

$$\Phi_{\text{max}} = 1$$

The fairness metric is minimal when the relative penalty values are maximally spread. In that case the fairness goes towards the value zero:

$$\lim_{\kappa \to 0} \Phi_{\text{min}} \to 0$$

The fairness metric is comprehended between the values zero and one:

$$\Phi \in [0....1]$$

G. The Equity Metric

The equity metric proposed in this paper evaluates the distribution of the cost penalty independent of the maximum penalty cost that has been defined by the airlines.

$$E = \left( \frac{\prod_{i=1}^{n} (P_i + e)}{\sum_{i=1}^{n} (P_i + e)} \right)^{\frac{1}{n}} \cdot n$$

where $e$ is a strictly positive value in order to ensure that when the cost penalty is zero for one flight of the set of $n$ flights, the equity does not result in zero. For example when for three flights there is no cost penalty, $P1=P2=P3=0$, the equity metric is maximal, as the same penalty, namely zero, has been distributed to all flights. If the value $e$ was not included in the metric, the equity metric would result to be zero.
It is important to notice that this metric provides a statement about the distribution of the incurred penalty costs and measures whether those have been distributed in an equal manner among all flights being analyzed, independent of the values defined for $P_{SAT}$.

Equity, as defined in section II, represents a special case of fairness. In this context, that case is given when for all flights considered, the preferences and the cost index are the same. The result is all flights having the same value for $P_{SAT}$. In such a situation, if one wants to distribute the additional incurred cost in the fairest manner, the result would be to distribute the cost penalties equally among all. Fairness is defined to be maximal when the values for the different relative penalties are the same. Thus, in this case, the fairest distribution is also the most equitable.

Similar to the fairness metric, the equity metric is maximal when the cost penalty has been equally distributed among all. In that case the equity is 1.

$$E_{\text{max}} = 1$$  \hspace{1cm} (29)

The equity metric is minimal when the cost penalty values have maximum dispersion. The value for the equity metric goes towards the value zero:

$$\lim_{e \to 0} E_{\text{min}} \rightarrow 0$$  \hspace{1cm} (30)

The equity metric is comprehended between the values zero and one:

$$E \in [0,1]$$  \hspace{1cm} (31)

**IV. Assessing the Performance of a SAS**

With a view to convey a better understanding of the proposed metrics described above, a closer look at certain particular cases is proposed and a discussion about the interpretation of these metrics and their informative value.

All cases exposed here are valid for three flights with the same aircraft type covering the same route between the same two city pair, with the same preferred flight time $T_P$ and fuel consumption $F_P$. All flights are executed during the same time period, and it is assumed that $C_F$ is equal for all flights.

- **Case A**: the preferred cost is respected for all three flights; no cost penalty is incurred in any of the three flights.

  - $\varphi_1 = \varphi_2 = \varphi_3 = 0$

  For this particular case, both metrics, $\Phi$ and $E$, are maximal because the preferences are fully respected, i.e. the preferred cost is achieved for all three flights:

  $$\Phi_A = \frac{\sqrt{(1-0) \cdot (1-0) \cdot (1-0)}}{1-0 + 1-0 + 1-0} \cdot \frac{3}{3} = 1$$

  In order to calculate $E$, we take advantage of the known relationship between the relative penalty and the cost penalty:

  $$E = \varphi_i \cdot \left( \frac{P_i}{P_{SAT} + \kappa} \right)$$

  For $\varphi_i = 0$, $P_i$ will be zero for any value of $P_{SAT}$. Therefore,

  $$E_A = \frac{\sqrt{(0+e) \cdot (0+e) \cdot (0+e)}}{(0+e) + (0+e) + (0+e)} \cdot \frac{3}{3 \cdot e} = 1$$
- **Case B**: for all three flights the percentage of the cost penalty incurred is the same positive value:

  \[ \phi_1 = \phi_2 = \phi_3 = \phi_B \text{ with } 0 < \phi_B < 1 \]

  In this case, the fairness with which the additional cost was distributed among the three flights is maximal:

  \[
  \Phi_B = \frac{3}{3} \left( \frac{1 - \phi_B}{1 - \phi_B} \right) \cdot \frac{1 - \phi_B}{1 - \phi_B} \cdot \frac{1 - \phi_B}{1 - \phi_B} \cdot 3 = \frac{1 - \phi_B}{3 - \phi_B} \cdot 3 = 1
  \]

  The value of the equity metric depends on the strategy of the different airlines. If for all flights the cost index is the same, this results in all flights having the same values for \( P_{SAT} \). In this case, the same amount of additional cost penalty was distributed among the three flights (\( P_1 = P_2 = P_3 = P_B \)), so the equity metric is maximal:

  \[
  E_{B1} = \frac{3}{3} \left( \frac{P + e}{P + e} \right) \cdot \frac{P + e}{P + e} \cdot \frac{P + e}{P + e} \cdot 3 = \frac{P + e}{3P + e} \cdot 3 = 1 \text{ for } CI_1 = CI_2 = CI_3
  \]

  If the value of the cost index is different for any of the flights, then the amount of additional cost penalty distributed among the three flights is not the same, resulting in a value for the equity metric less than 1.

  Consider the case where \( CI_1 = CI_2 < CI_3 \). From the reasoning above it can be concluded that \( P_1 = P_2 = P_3 = P_B \). According to the definition for \( P_{SAT} \) below, it can be easily shown that for all other parameters assumed to be fixed, \( P_{SAT} \) grows as the cost index \( CI \) grows.

  \[
  P_{SAT} = C_F \sqrt{CI^2 \left( T_{REF} - T_p \right)^2 + \left( F_{REF} - F_p \right)^2}
  \]

  Therefore, knowing that \( P_{SAT1} \) is larger than \( P_{SAT2} \), the value for \( P_3 \) must be higher than \( P_1 \) (or \( P_2 \)) maintaining the same proportion \( \phi_B \). As a result,

  \[
  E_{B2} = \frac{3}{2 \cdot P + P_3 + 3 \cdot e} < 1
  \]

  Recalling the definition of the equity metric, the geometric mean in the numerator is smaller than the arithmetic mean in the denominator for any set of non-identical (positive) values, which leads to the inequality just above.

- **Case C**: for all three flights the maximum cost penalty is given:

  \[ \phi_1 = \phi_2 = \phi_3 = \phi_C \text{ where } \phi_C \approx 1 \text{ for } \kappa \to 0 \]

  Independent of each value for the maximum cost penalty, the fairness metric is maximal, because the three flights have to incur the same relative penalty:

  \[
  \Phi_C = \frac{3}{3} \left( \frac{1 - \phi_C}{1 - \phi_C} \right) \cdot \frac{1 - \phi_C}{1 - \phi_C} \cdot \frac{1 - \phi_C}{1 - \phi_C} \cdot 3 = 1
  \]

  The value of the equity metric depends again on the different airline strategies. If the three flights have the same cost index value, then for all flights the maximum cost penalty is the same:

  \[ P_{SAT1} = P_{SAT2} = P_{SAT3} = P_{SAT} \]

  In that case, the equity metric is maximal because the same cost penalty was distributed to the three flights:
For different airline strategies, that means for different values of the cost index, the equity metric results in a value less than 1. Considering again the case where CI_1 = CI_2 < CI_3, it can be concluded that P_{SAT_1} = P_{SAT_2} (= P_{SAT}) and that P_{SAT_3} is larger than P_{SAT_1} (or P_{SAT_2}). This results in the equity metric:

\[ E_{C2} = \frac{3}{3} \left( \frac{(P_{SAT} + e) \cdot (P_{SAT} + e) \cdot (P_{SAT_3} + e)}{(P_{SAT} + e) + (P_{SAT} + e) + (P_{SAT_3} + e)} \right) \cdot 3 < 1 \]

Due to the same reasoning as for \( E_{B2} \), the above inequality applies for \( E_{C2} \). Additionally, we can deduce that \( E_{B2} = E_{C2} \) only if the cost indexes (CI_1, CI_2 and CI_3) are the same for both cases.

**Case the strategies differ:** for all three flights, different cost penalties are given. The preferred flight cost is respected for flight one, flight two reaches the maximum cost penalty and flight three accommodates a positive value P_D.

- \( P_1 = 0 \); \( P_2 = P_{SAT_2} \); \( P_3 = P_D \) where \( 0 \leq P_D < P_{SAT_3} \)

The relative penalties result to be all different:

\[ \phi_1 = 0 \); \( \phi_2 \approx 1 \) for \( \kappa \rightarrow 0 \); \( \phi_3 = \phi_D \)

Thus, the fairness metric results in value smaller than 1:

\[ \Phi_D = \frac{\sqrt{1 \cdot (1 - \phi_2) \cdot (1 - \phi_D)}}{1 + (1 - \phi_2) + (1 - \phi_D)} \cdot 3 < 1 \]

Independently of the airline strategy, the equity metric also results in a value smaller than 1:

\[ E_D = \frac{3}{3} \left( \frac{e \cdot (P_{SAT_2} + e) \cdot (P_D + e)}{e + (P_{SAT_2} + e) + (P_D + e)} \right) \cdot 3 < 1 \]

**Case E:** all three flights have the same cost penalty:

- \( P_1 = P_2 = P_3 = P_E \) for \( 0 < P_E \leq P_{SAT} \)

Thus, the equity metric is maximal:

\[ E_E = \frac{3}{3} \left( \frac{(P_E + e) \cdot (P_E + e) \cdot (P_E + e)}{(P_E + e) + (P_E + e) + (P_E + e)} \right) \cdot 3 = \frac{P_E + e}{3(P_E + e)} \cdot 3 = 1 \]

Depending on the airline strategy, the fairness metric adopts different values. If all flights have the same strategy, then the cost index value is the same for all flights. All flights result to have the same value for P_{SAT}, thus the incurred relative cost penalty is the same for the three flights (\( \phi_1 = \phi_2 = \phi_3 = \phi_E \)). Then, the fairness is maximal:

\[ \Phi_{E1} = \frac{3}{3} \left( \frac{(1 - \phi_E) \cdot (1 - \phi_E) \cdot (1 - \phi_E)}{(1 - \phi_E) + (1 - \phi_E) + (1 - \phi_E)} \right) \cdot 3 = 1 \] for CI_1 = CI_2 = CI_3

Are the strategies different, then the cost index values are different and the fairness metric results in a value between 0 and 1. Consider again the case where CI_1 = CI_2 < CI_3, implying that results in P_{SAT_1} = P_{SAT_2} (= P_{SAT}) and P_{SAT_3}

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being larger than \( P_{\text{SAT}1} \) (or \( P_{\text{SAT}2} \)). As a result the relative penalty for flights one and two are identical and greater than the penalty percentage of flight three (\( \varphi_1 = \varphi_2 = \varphi > \varphi_3 \)).

\[
\Phi_{E2} = \frac{3}{(1 - \varphi)} \cdot (1 - \varphi) \cdot (1 - \varphi_3) \cdot \frac{3}{1 - \varphi_3} < 1
\]

- **Case F:** for all three flights, different cost penalties are given. The preferred flight cost is respected for flight one, flight two and three reach their maximum cost penalty:
  - \( P_1 = 0; \ P_2 = P_{\text{SAT}2}; \ P_3 = P_{\text{SAT}3} \)

  In this case, independent of the strategy of each flight, the equity and the fairness metric result in a value between 0 and 1:
  \( \varphi_1 = 0; \ \varphi_2 = \varphi_3 = \varphi_f \approx 1 \) for \( \kappa \rightarrow 0 \)

\[
\Phi_F = \frac{3}{1 + (1 - \varphi_f) + (1 - \varphi_f)} \cdot \frac{3}{1 - \varphi_f} < 1
\]

\[
E_F = \frac{3}{e \cdot (P_{\text{SAT}2} + e) \cdot (P_{\text{SAT}3} + e)} \cdot \frac{3}{e + (P_{\text{SAT}2} + e) + (P_{\text{SAT}3} + e)} < 1
\]

**Case G:** for all three flights, different cost penalties are given. The preferred flight cost is respected for flight one and flight two while flight three reaches its maximum cost penalty:
  - \( P_1 = P_2 = 0; \ P_3 = P_{\text{SAT}3} \)

  In this case is similar to case F. Independent of the strategy of each flight, the equity and the fairness metric result in a value between 0 and 1:
  \( \varphi_1 = \varphi_2 = 0; \ \varphi_3 \approx 1 \) for \( \kappa \rightarrow 0 \)

\[
\Phi_G = \frac{3}{1 + (1 - \varphi_3)} < 1
\]

\[
E_G = \frac{3}{e \cdot (P_{\text{SAT}3} + e)} \cdot \frac{3}{e + (P_{\text{SAT}3} + e)} < 1
\]

Case F and Case G are similar but the comparison of both cases, assuming \( P_{\text{SAT}3} \) has the same value in both cases (\( \varphi_3 = \varphi_f \) in both cases), shows that the fairness and equity metric have different properties.

Both penalize the dispersion of the cost penalty but the fairness metric, per definition, penalizes not only the dispersion of the values but also the proximity to their maximum cost penalties defined by the airlines.

Recalling the definition of the fairness metric, the metric is based on the relative penalty. The relative penalty evaluates how far the cost penalty incurred is from its maximum value. The fairness metric incorporates the expression \((1 - \varphi)\) which reflects the airline satisfaction. That expression penalizes the relative penalty values that approach their maximum value, namely one. That is the case when the cost penalty incurred is equal to the maximum cost penalty defined by the airline. Thus, the fairness metric penalizes the proximity of the cost penalty values to their maximum cost penalty \( P_{\text{SAT}} \). According to this \( \Phi_F < \Phi_G \):
\[
\frac{3\sqrt{(1 - \varphi_F)^2}}{3 - 2\varphi_F} \cdot 3 < \frac{3\sqrt{(1 - \varphi_3)^2}}{1 + 1 + (1 - \varphi_3)} \cdot 3 \quad \text{where } \varphi_3 = \varphi_F
\]

\[
\frac{3\sqrt{(1 - \varphi_3)^2}}{3 - 2\varphi_3} < \frac{3\sqrt{(1 - \varphi_3)^2}}{3 - \varphi_3}
\]

\[
\frac{3\sqrt{(1 - \varphi_3)}}{3 - \varphi_3} < \frac{3 - 2\varphi_3}{3 - \varphi_3} \quad \text{where } \varphi_3 \approx 1 \text{ for } \kappa \to 0
\]

Focusing now on the equity metric, the equity metric E does not take into consideration the values of \(P_{SAT}\) and therefore the maximum penalty defined by the airlines. The equity metric evaluates the distribution of the absolute cost penalty value P. This is an interesting example to show how the equity and fairness metric differ. While the fairness metric penalizes the proximity of the cost penalty to the corresponding value of \(P_{SAT}\) defined by the airline, the equity metric, as it does not consider \(P_{SAT}\), does not penalize that proximity.

V. Conclusions

In order to measure the fairness or equity of a system, a just framework has to be provided. This document describes at the beginning of section II the requirements to ensure a just framework, namely to agree upon certain standards by those whom they apply to, under conditions preventing them from tailoring the principles to their own advantage.

Within a just framework, the fairness and equity can be evaluated. Before that, fairness and equity have to be defined. This paper provides a clear definition of the concept of fairness and equity in ATM.

According to the requirements of SESAR and NextGen concept of operations for the future trajectory based operation environment, a cost model is proposed for the development of the fairness and equity metrics. This cost model is based on assumptions already regarded within SESAR and NextGen.

The proposed cost model and cost penalty model present a comprehensive set of assumptions and constraints in order to define a cost penalty function as well as a relative cost penalty function. An example of a cost penalty function and a relative cost penalty function is provided in this work.

Nevertheless, it has to be clear that the cost model, as well as the functions and the developed metrics, have been defined in a generic way. The examples provided for the cost penalty as well as for the relative cost penalty function are intended to help the metric development. Although the metrics are based on these functions, both metrics are independent of the possible expressions of the cost penalty function and of the relative cost function.

The detailed fairness metric fully complies with the definition provided for the concept of fairness in ATM. The metrics captures the three main elements:

- achieve a balance of conflicting interest by means of a just procedure (integrated in the constraints of the cost function and the CI)
- take into account the acceptance levels of a society and satisfaction of individuals (through the definition of \(T_{REF}\) and \(F_{REF}\), which lead to \(P_{SAT}\))
- is inherently relational (integrated in the metric definition)

The fairness metric is based on the relative penalty, which shows the cost penalty incurred with respect to the maximum acceptable cost penalty defined by the airline. The maximum acceptable cost penalty is defined by the airlines according to its cost strategy for a certain flight. This is an instrument for the airlines in order to express its acceptance levels.

Fairness is based on the honesty of the individuals when expressing their acceptance levels and satisfaction. If the individuals, in this case the airlines, are untruthful, then the correct evaluation of fairness cannot be guaranteed. In those cases, the best solution is to evaluate the equity of the system and make decisions upon the results of the equity metric.

The equity metric is based on the cost penalty function, thus evaluating the distribution of the absolute cost penalty values and does not take into account the acceptance levels of the airlines. As long as a just framework is provided, equity can be measured.

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This paper starts at the conceptual definition of fairness and equity in ATM. This is important as most papers dealing with fairness or equity do not clearly define those concepts.\textsuperscript{14,15} This paper also defines a consistent cost model to develop the metrics which evaluate fairness and equity capturing all elements of the conceptual definition. With the help of different cases used for the analysis of the metrics, the consistency of the metrics with the concept definition is confirmed.

The resulting metrics are intended to build the basis for the extrapolation to a standard methodology for evaluating the fairness and equity of different types of systems modifying the trajectories while trying to accommodate the user preferences. Such systems are for example Decision Support Tools (DST) to be implemented in different ATM situations (e.g. flow management tools).

Assessing the fairness and equity of DSTs, as for example the ones assisting air traffic controllers in the separation assurance activities, is key for the successful implementation of those automated systems in future ATM system.

References

\textsuperscript{1} Breunig, Tamara, Steve Bradford, and Diana Liang, “Standardizing Performance Metrics,” 5\textsuperscript{th} USA/Europe Air Traffic Management R&D Seminar, June 2003
\textsuperscript{2} Civil Air Navigation Services Organisation (CANSO) URL: www.canso.org [cited June 2009]
\textsuperscript{3} International Air Transportation Association (IATA) URL: www.iata.org [cited June 2009]
\textsuperscript{4} International Civil Aviation Organisation (ICAO) URL: www.icao.int [cited June 2009]
\textsuperscript{5} Wierzbicka, Anna, \textit{English: Meaning and Culture}, Chapter 5 “Being Fair Another Key Anglo Value and Its Cultural Underpinnings,” Oxford University Press, New York, April 2006
\textsuperscript{11} Audi, Robert, \textit{The Cambridge Dictionary of Philosophy}, Cambridge University Press, Cambridge, UK, September 1999
\textsuperscript{12} Manzi, Patrick, Lars Lindberg, Hanyo Vera Anders, Per Ahl, Bengt Nilsson, and Francisco A. Navarro, “COURAGE UPTs in Unconstrained ATM Environment,” \textit{Eurocontrol COURAGE Deliverable D1.1}, 13 June 2005
\textsuperscript{13} Airbus Costumer Services, “Flight Operations Support & Line Assistance, Getting to Grips with the Cost Index,” STL 945.2369/96, Issue II- May 1998