Adaptive Disturbance Tracking Control for Large Horizontal Axis Wind Turbines in Variable Speed Region II Operation

Mark J. Balas
University of Wyoming, Laramie, WY, 82071

Qian Li
University of Wyoming, Laramie, WY, 82071

and

Ryan Peterman
University of Wyoming, Laramie, WY, 82071

A new control problem called Disturbance Tracking Control (DTC), which arises in active control of variable speed horizontal axis wind turbines for electric power generation, was developed previously. Feedback control of a linear plant, which is persistently disturbed, must cause the plant output to track a linear function of the disturbance. This control theory is related to Tip Speed Ratio Tracking for wind turbines operating in Region II.

However, the DTC approach was developed for fixed gain controllers where the parameters of the turbine are very well known. In this paper we present an adaptive version of the DTC Theory for turbines with poorly known parameters. We apply this new theory to create an adaptive tip speed ratio tracking controller for a horizontal axis wind turbine generator based upon a model of the NREL Controls Advanced Research Turbine (CART).

Nomenclature

- \( R \) = radius of the turbine blades
- \( \lambda \) = Tip-speed ratio
- \( \lambda_{op} \) = Desired operating point of Tip-speed ratio
- \( \omega \) = Hub-height wind speed
- \( \omega_T \) = Turbine rotor angular speed
- \( \omega_{op} \) = Desired operating point of Hub-height wind speed
- \( \omega_T^{op} \) = Desired operating point of Turbine rotor angular speed
- \( u \) = Linear plant control inputs
- \( u_d \) = Linear plant disturbance input(s)
- \( x_p \) = Linear plant states
- \( y_p \) = Linear plant outputs
- \( e_y \) = Output Tracking Error
- \( G_e, G_D \) = Adaptive gain matrices of the appropriate compatible dimensions
- \( \gamma_e, \gamma_D \) = Arbitrary, positive definite matrices

---

1 Professor and Department Head, Electrical and Computer Engineering, AIAA Fellow.
2 Graduate Student, Electrical and Computer Engineering
3 Graduate Student, Electrical and Computer Engineering
I. Introduction

Large wind turbines are operated in three regions. Region I is the turbine start up; Region III is the turbine operated at rated power, and Region II is the turbine operating in between Regions I and III with enough wind to generate power but not at full rated capacity. In Region III blade pitch control is used to achieve constant rotor speed, and in region II generator torque is controlled via the power electronics to produce a constant tip speed ratio. This paper will focus entirely on Region II operation where a constant tip speed ratio produces the best results.

We present a new theory of Adaptive Disturbance Tracking Control (DTC) and apply it to the Region II control problem for Horizontal Axis Wind Turbine (HAWT) electric power generation. Simulation results will be presented on a dynamic model of the NREL Controls Advanced Research Turbine (CART).

An approach to the reduction or counteraction of persistent disturbances was developed by Johnson in [1]. In [2], Balas developed the idea of disturbance tracking control, and Stol modified this in [3]. The motivation for DTC comes from the desire to use control to track some of the wind disturbance. Performance of the wind turbine in Region II is dependent upon the Tip Speed Ratio (TSR):

$$\lambda \equiv \frac{\omega_r R}{\omega}$$

where $\omega$ is the wind speed flowing into the turbine blades, $\omega_r$ is the turbine rotor angular speed, R is the radius of the turbine blades. Best performance in Region II is obtained by keeping this TSR constant.

We actually linearize the TSR Error:

$$\varepsilon \equiv \lambda - \lambda_{op} = \frac{\omega_r R}{\omega} - \frac{\omega_r^{op} R}{\omega_{op}} \approx \frac{R}{\omega_{op}} [\Delta \omega_r - \omega_r^{op} \Delta \omega]$$

(1)

Where $\Delta \omega_r \equiv \omega_r - \omega_r^{op}$, $\Delta \omega \equiv \omega - \omega_{op}$, and $(\omega_r^{op}, \omega_{op})$ is the desired turbine operating point corresponding to the desired TSR $\lambda_{op}$. We let the Output Tracking Error be

$$e_y \equiv \Delta \omega_r - Q \Delta \omega$$

with $Q \equiv \frac{\omega_r^{op}}{\omega_{op}}$ (2)

and think of $\Delta \omega_r$, the turbine speed variation, as a measured output of the turbine and $\Delta \omega$, the wind speed fluctuations, as a disturbance on the turbine. Then DTC becomes choosing a feedback control law that produces:

$$e_y \equiv \Delta \omega_r - Q \Delta \omega \xrightarrow{t \rightarrow \infty} 0$$

(3)

This approximately produces tracking of the desired TSR: $\varepsilon \equiv \lambda - \lambda_{op} \xrightarrow{t \rightarrow \infty} 0$.

II. Adaptive Disturbance Tracking Control Theory with Persistent Disturbances

The design of the Region II adaptive torque controller makes use of a direct adaptive control approach with adaptive tracking of persistent disturbances. In this section we develop the general adaptive DTC theory. In [4], we presented an adaptive approach to turbine speed regulation for Region III using similar ideas, but the overall results are different for adaptive DTC.

The plant is assumed to be well modeled by the linear, time-invariant, finite-dimensional system:

$$\begin{align*}
\dot{x}_p &= Ax_p + Bu_p + Gu_D \\
y_p &= Cx_p;
\end{align*}$$

(4)

where the plant state, $x_p(t)$, is an $N_p$-dimensional vector, the control input vector, $u_p(t)$, is $M$-dimensional, and the sensor output vector, $y_p(t)$, is $P$-dimensional. The disturbance input vector, $u_D(t)$, is $M_D$-dimensional and will be thought to come from the Disturbance Generator.
where the disturbance state, $z_D(t)$, is $N_D$-dimensional. All matrices in Eqs. (4)-(5) have the appropriate compatible dimensions. Such descriptions of persistent disturbances were first used in [1] to describe signals of known form but unknown amplitude. Equation (5) can be rewritten in a form that is not a dynamical system, which is sometimes easier to use:

$$
\begin{align*}
\mathbf{u}_D &= \Theta \mathbf{z}_D \\
\dot{\mathbf{z}}_D &= \mathbf{F} \mathbf{z}_D; \mathbf{z}_D(0) = \mathbf{z}_0
\end{align*}
$$

(6)

where $\phi_D$ is a vector composed of the known basis functions for the solution of $\mathbf{u}_D = \Theta \mathbf{z}_D$, i.e., $\phi_D$ are the basis functions which make up the known form of the disturbance, and $L$ is a matrix of appropriate dimension. The method for tracking persistent disturbances used in this paper requires only the knowledge of the form of the disturbance, the amplitude of the disturbance does not need to be known, i.e. $(\mathbf{L}, \Theta)$ can be unknown. In this paper, we will be interested in rejecting step disturbances of unknown amplitude which can be represented in the form of Eq. (6) as $\phi_D \equiv \mathbf{1}$, with $(\mathbf{L}, \Theta)$ unknown. This has been a viable model for wind fluctuations in our previous work.

Our control objective will be to cause the output of the plant, $y_p(t)$, to asymptotically track some linear function of disturbances of the form given by the disturbance generator. We define the output tracking error vector as:

$$
e_y \equiv y_p - Qu_D
$$

(7)

To achieve the desired control objective, we want $e_y \underset{t \to \infty}{\to} \Theta$. This aligns with the previous discussion for wind turbine Region II operation in Section 1.

Consider the plant given by Eq. (4) with the disturbance generator given by Eq. (5). Our control objective for this system will be accomplished by an Adaptive Control Law of the form:

$$
\mathbf{u}_p = \mathbf{G}_e e_y + \mathbf{G}_D \phi_D
$$

(8)

where $\mathbf{G}_e$ and $\mathbf{G}_D$ are adaptive gain matrices of the appropriate compatible dimensions.

Now we specify the Adaptive Gain Laws, which will produce asymptotic tracking:

$$
\begin{align*}
\dot{\mathbf{G}}_e &= -e_y e_y^T \gamma_e \\
\dot{\mathbf{G}}_D &= -e_y \phi_D^T \gamma_D
\end{align*}
$$

(9)

where $\gamma_e, \gamma_D$ are arbitrary, positive definite matrices. Our Adaptive Controller is specified by Eq. (8) with the above adaptive gain laws Eq. (9). In the next section, we analyze the stability of this controller and showed that the adaptive gains, $\mathbf{G}_e$ and $\mathbf{G}_D$, remain bounded and asymptotic tracking occurs, i.e., $e_y \underset{t \to \infty}{\to} \Theta$. However, it is important to note that we must measure $e_y \equiv y_p - Qu_D$ in order to implement the adaptive gains! But this requires some measurement of $u_D$ which was not required in [2]-[3] or in the Region III adaptive control in [4].
III. Stability Analysis of Adaptive Disturbance Tracking Control Theory

We begin with the idea of Ideal Trajectories as in [2]:

\[
\begin{align*}
\dot{x}_s &= Ax_s + Bu_s + \Gamma u_D \\
y_s &= Cx_s = Qu_D
\end{align*}
\]

with
\[
\begin{align*}
x_s &= S_1^* z_D \\
u_s &= S_2^* z_D
\end{align*}
\]

This just means that the Plant (4) can actually accomplish the desired output tracking. It is a theoretical construct and the ideal trajectories are never needed in the implementation of Adaptive DTC!

It is easy to see that (10) is equivalent to the following Matrix Matching Conditions:

\[
\begin{align*}
S_1^* F &= AS_1^* + BS_2^* + \Gamma \theta \\
CS_1^* &= Q \theta
\end{align*}
\]

It is well known [5] that unique gains \((S_1^*, S_2^*)\) can be found to solve (11) when CB is nonsingular, but again they will never be needed for the controller implementation. We form:

\[
\begin{align*}
\Delta x &= x_p - x_s \\
\Delta u &= u_p - u_s \\
\Delta y &= y_p - y_s = y_p - Qu_D \equiv e_y
\end{align*}
\]

and obtain:

\[
\begin{align*}
\dot{\Delta x} &= A \Delta x + B \Delta u \\
\Delta y &= C \Delta x = e_y
\end{align*}
\]

From (8), we have

\[
u \equiv G_c e_y + G_D \phi_D \Rightarrow \Delta u \equiv u - u_s = [G_c e_y + G_D \phi_D] - [S_2^* L] \phi_D = G_c^* e_y + \Delta G \eta
\]

where

\[
G \equiv G - G_c = \begin{bmatrix} \Delta G_c & \Delta G_D \end{bmatrix}
\]

\[
\eta \equiv \begin{bmatrix} e_y \\ \phi_D \end{bmatrix}
\]

Then

\[
\begin{align*}
\dot{\Delta x} &= A_c \Delta x + Bw; w \equiv \Delta G \eta \\
\Delta y &= C \Delta x = e_y
\end{align*}
\]

with \(A_c \equiv A + BG_c^* C\)

\[
\dot{G} = \Delta \dot{G} = -e_y \eta^T \gamma; \gamma \equiv \begin{bmatrix} \gamma_c & 0 \\ 0 & \gamma_D \end{bmatrix} > 0 \text{ or }
\]

And, from (9),

\[
\begin{align*}
\dot{G}_c &= \Delta \dot{G}_c = -e_y^T e_y \\
\dot{G}_D &= \Delta \dot{G}_D = -e_y^T \phi_D \gamma_D
\end{align*}
\]

We prove the following Stability Result:
**Theorem 1:** Let \( (A_p, B_p, C_p) \) be Almost Strictly Positive Real (ASPR) or, equivalently, \( C_p B_p > 0 \) and \( P(s) \equiv C_p(sI - A_p)^{-1}B_p \) minimum phase and \( \phi_D \) bounded. Then the Adaptive DTC Controller in (8)-(9) produces \( e_y = y_p - Qu_p \xrightarrow{t \to \infty} 0 \) with bounded adaptive gains \( (G_e, G_D) \).

**Proof:** Define \( V(\Delta x) \equiv \frac{1}{2} \Delta x^T P \Delta x \) with \( P > 0 \) \( \exists \begin{cases} A_c^T P + PA_c = -Q < 0 \\ PB = C^T \end{cases} \). This follows from \( P(s) = C(sI - A)^{-1}B \) ASPR.

Then \( \dot{V}(\Delta x) = -\frac{1}{2} \Delta x^T Q \Delta x + \langle e_y, w \rangle \)

Define \( V(\Delta G) = \frac{1}{2} tr(\Delta G \gamma^{-1} \Delta G^T) \Rightarrow \dot{V}(\Delta G) = -\langle e_y, w \rangle \). Therefore, \( V \equiv V(\Delta x) + V(\Delta G) \Rightarrow \dot{V} = -W(\Delta x) = -\frac{1}{2} \Delta x^T Q \Delta x \leq 0 \) and all trajectories \( (\Delta x, \Delta G) \) are bounded. From Barbalat’s Lemma applied to \( W(\Delta x) \), we will have \( W(\Delta x) \xrightarrow{t \to \infty} 0 \) which leads to \( \Delta x \xrightarrow{t \to \infty} 0 \) because of positivity of \( W(\Delta x) \), and \( e_y = \Delta y = C\Delta x \xrightarrow{t \to \infty} 0 \), as desired.

**Matching Conditions Special Case:**
We consider one special case for the Matching Conditions in (11).

\[
\begin{align*}
A & = \begin{bmatrix} 0 & I \\ A_{21} & A_{22} \end{bmatrix}; B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}; C = \begin{bmatrix} 0 & C_1 & C_2 \end{bmatrix}; F = 0 \\
C_1 S_A + B_2 S_2^* + \Gamma_2 \theta &= 0. \\
C_1 S_A &= Q \theta
\end{align*}
\]

Since \( P(s) \) is ASPR, we have \( CB = C_2 B_2 > 0 \); so \( C_2 B_2 S_2^* = -C_2 [A_{21} S_A + \Gamma_2 \theta] \) or \( S_2^* = -(C_2 B_2)^{-1} C_2 [A_{21} S_A + \Gamma_2 \theta] \) and we find \( S_1^* = \begin{bmatrix} 0 \\ S_A \end{bmatrix} \) from \( C_1 S_A = Q \theta \). This special case is the usual form of the linear dynamics of the plant (4) for wind turbines.

As an illustrative

\[
\begin{align*}
A &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; C = \begin{bmatrix} 1 & 0.1 \end{bmatrix}; F = 0; \theta = 1; Q = 0.5 \\
S_1^* &= \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} \quad \text{and} \quad S_2^* = -1
\end{align*}
\]

\( CB = C_2 B_2 = 0.1 > 0 \) and \( P(s) = \frac{1 + 0.1 s}{s^2} \) is minimum phase; so \( P(s) \) is ASPR. This example illustrates the adaptive DTC operation in theory and in simulation. But we apply the result of Theo 1 to a much more serious wind turbine example in the next section.
IV. Application of Adaptive Disturbance Tracking Control Theory to the NREL (National Renewable Energy Laboratory) CART (Control Advanced Research Turbine)

A follow-up goal outside of region 3 has been a desire to design a DTC (Disturbance Tracking Control) model in region 2. This design will drive the error (a function of wind speed and turbine rotor speed) to 0 as time approaches infinity. In region 3, because the generator is already delivering maximum output power, it isn’t desired to adapt its operation, which is why the blade pitch is changed based upon the wind input disturbance. However, in region 2, the generator is not delivering maximum power. With this in mind, it is able to develop a torque controller for the generator that is controlled by a tip speed ratio from the blades on the turbine. It is the objective of this controller to maximize power output and efficiency in operating region 2 of the turbine.

The turbine for modeling is called the Controls Advanced Research Turbine (CART) used in NREL (National Renewable Energy Laboratory). It is a 600 kW upwind machine with two-bladed, teetered rotor has a radius of $R=21.3$ m. Control inputs include generator torque and individual blade pitch. In region 2, we keep the blade pitch angle as a constant, and only control generator torque.

![Figure 1. Simulink Adaptive Disturbance Tracking Control Model](image)

The interior of the torque controller for the original DTC model is not significant as it currently operates in all the operating regions of the turbine. Because we are only dealing with one region of the turbine (region 2) we will install the adaptive DTC controller in place of the existing interior of the torque controller. The interior of the torque controller is shown in Figure 2. We can change the gain $\gamma_e$ and $\gamma_D$ to adjust the performance of output just like the overshoot and Settling Time. The actual values of the gains used here in the adaptive controller are: $\gamma_e = 0.015$, $\gamma_D = 13.5$. 

6
For application purposes, simulations were run with this model, as well as the original model in order to compare the output data and mechanical response of the system with both the original DTC model and the adaptive DTC model. Figure 3 below is a plot of the wind file we used as a disturbance for simulations on both models in order to obtain consistency with our test runs. Here $u_{op} \approx 2575$.

To test this design we will first test on step wind, then on turbulent wind.

**A. Step wind**

Figure 3 is a plot of a wind input file as a function of time that is step wind only in Region 2. Using this wind file, we compare torque command from the torque controller, torque command rate, and tip speed ratio on both models.
We can see the performance of original DTC controller and the adaptive DTC controller are very close. But the torque commands differ by about 25N*m.

We calculate the Root Mean Square Error between the torque commands of the Original DTC and the Adaptive DTC controller to be about 136.25 N*m using step wind.
We can see from above that the torque rate of the Original DTC controller is larger than the Adaptive DTC controller. That means the Adaptive DTC controller changes more smoothly than the original DTC.
The tip speed ratios for both controllers are essentially the same in Figure 8.

B. Turbulent wind
Figure 9 is a plot of a turbulent wind input file in Region 2. We also compare torque command from the torque controller, torque command rate, and tip speed ratio on both models as following.

![Figure10. Torque Command of Turbulent Wind](image)

![Figure11. Zoom-in of Torque Command of Turbulent Wind](image)
We also calculate the Root Mean Square Error between the torque commands of those two controllers to be about 137.19 N*m using turbulent wind.

Figure 12. Torque Command Rate of Turbulent Wind

Figure 13. Zoom-in of Torque command Rate of Turbulent Wind
The tip speed ratios for both controllers here are also the same using turbulent wind in Figure 14.

V. Conclusion

In this paper we have developed a new theory of Adaptive Disturbance tracking control which extends the fixed gain DTC theory of [2]-[3]. It is easy to see that the adaptive version of DTC in (8)-(9) is vastly simpler than the fixed gain version. However, at present, the adaptive version requires measurement of a linear function of the wind speed.

Numerical results on a simulation of the NREL CART Turbine show the promise of adaptive DTC in tracking a desired Tip Speed Ratio in Region II operation. The adaptive DTC does a remarkably good job of replicating the torque commands of the fixed gain DTC with no increase in actuator rates.

Acknowledgements

This work is supported by Wind Energy Research Center at the University of Wyoming, and we thank Dr. Alan Wright of National Renewable Energy Laboratory for many helpful suggestions.

References