An Evaluation of Common Analysis Methods for Bolted Joints in Launch Vehicles

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Accurate calculation of bolt force in a bolted joint under external loads and temperature changes is a fundamental requirement in many industries. Approximate hand formulas, based on idealized mechanical models, are widely used. We evaluate the accuracy of several hand formulas by investigating the consistency of the underlying models and by comparing hand predictions to detailed finite element results. We exploit finite element predictions for the differential thermal expansion problem to provide estimates of effective geometric quantities uncontaminated by the prying action introduced by external forces. We then use these accurately determined effective quantities to make hand predictions of bolt force in a flange joint under combined external loading and temperature changes. We compare these hand predictions to results from a second detailed finite element model of the flange joint. Finally, we use the flange joint model to gain insight into the joint softening that arises from gradual, nonlinear opening of the flange gap under external tension.

I. Introduction

Structures in aerospace, energy and industrial applications are often connected by bolted flange joints. One common application is in joining major substructures of launch vehicles. The capability of these joints is determined by analysis, augmented by testing. Often, the bolts and flanges are of different materials, so differential thermal expansion, as well as external loads, can load the bolt. A basic objective of bolted joint design is to provide adequate joint strength and stiffness while minimizing the fluctuating stress in the bolt induced by external loads and temperatures. Fluctuating bolt stress is the direct cause of bolt fatigue, which is a significant and costly failure mode. Therefore, it is vital that bolt loads be accurately calculated for realistic design and service conditions.

Bolted joints are frequently analyzed using hand formulas that embody many significant assumptions. In this paper, we evaluate several commonly used formulas by comparing their predictions to those of detailed finite element (FE) models. Our models allow the relaxation of many assumptions and enable a rational appraisal of these very important and widely used formulas. In addition, the FE results provide an understanding of the mechanics, which is just as important as the ability to make accurate predictions in specific cases.

There is no shortage of recent experimental and FE studies of bolted joint behavior. Wileman et al.1 used FE analysis to make numerical comparisons to some of the hand formulas we consider here. However, they used a rigid fastener head bonded to the clamped member, in order to prevent washer behavior from influencing the member stiffness, whereas we would argue that member stiffness in fact does depend on fastener head behavior in a real joint. Grosse and Mitchell2

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showed that gradual member separation under external loads leads to significantly nonlinear joint stiffness, rather than linear behavior followed by sudden gapping. They observed that external loads can create significant prying even in an axisymmetric problem. Lehnhoff et al. showed that when clearance exists, hand calculations would be improved by distinguishing between the hole diameter and the bolt diameter, and they suggested other improvements in the calculation of member stiffness.

Lehnhoff, with other collaborators, investigated in more detail the gradual member separation noted by Grosse and Mitchell, and later showed that accounting for the fastener threads in FE models, which was done approximately by Grosse and Mitchell, was necessary to obtain accurate stiffness predictions. Pedersen and Pedersen showed that Poisson’s ratio of the clamped material has a significant effect on its axial stiffness, and they implemented an energy-based method of extracting the member stiffness. Nassar and Abboud extended existing hand formulas to account for unequal clamped member thicknesses and moduli, and nonstandard bolt head-to-shank ratios. They augmented their analysis with experimental data and FE modeling.

Bolted joints are deceptively complex, and different designs show significantly different behavior. The quantitative results obtained here are applicable to one type of design widely used in many fields, but not necessarily to other designs. The aspects that distinguish our study are (i) recognition that a careful definition of bolt elongation is demanded by the generally assumed one-dimensional (1-D) mechanical model, (ii) consideration of an unconstrained thermal expansion problem, both for its relevance to design and because it is closer to 1-D than external loading situations are, (iii) extension of 1-D findings to a realistic flange joint, and (iv) use of state-of-the-art FE models including threads, elastic-plastic fasteners, frictional contact and prying.

In this study, we first analyze an axisymmetric thermal stress model with no external loads or constraints, both for comparison to hand formulas and to gain a detailed understanding of the stiffness relationships and joint behavior. Then, we analyze a bolt circle joining two flanges, to predict bolt load and joint stiffness for loads ranging up to the joint failure load. Differential thermal expansion is included. We compare the results from both analyses to those obtained from commonly used formulas.

Using a detailed FE analysis as a surrogate for testing has advantages and disadvantages. The disadvantage is of course that the analysis may lack, or inadequately resolve, some significant effect present in the real hardware. The advantages are that the boundary conditions, material properties, geometry and loads can be precisely controlled, interesting quantities that may be impractical to measure can be recovered easily, and it is less expensive to obtain the insight that comes from observing a very large number of cases than it would be if testing were the sole approach.

II. Thermal Stress in Absence of External Load or Constraint

A. Introduction

Bolted joints may be exposed to temperature changes of hundreds of degrees, as well as external loads. A change in temperature after fastener installation can induce significant stress when the bolt has a different coefficient of thermal expansion from the flanges.

The design to be considered here is a steel bolt-nut-washer set joining two flanged shells on a circular pattern (Figure 1). However, first, the basic stiffness relationships between the fastener and the clamped material must be understood. The offset distance from the bolt centerline to the shell creates prying that obscures these basic relationships. External applied or constraint loads transmitted to the joint through the shells act at a distance from the bolt circle, tending to pry open the flanges rather than directly lifting them off one another. Even if the external load is aligned with the bolt axis using a fitting, the flanges will peel apart instead of gapping all at once,
This section has two purposes:

- to assess whether popular hand-calculation formulas for bolt and flange stiffness can be used to accurately predict the bolt load change due to thermal expansion, and
- to gain insight into the joint mechanics when loads and displacements are perfectly axisymmetric, so that the lessons can be applied to the more realistic joint design shown in Figure 1.

The second purpose is arguably the more important, because bolt load changes due to thermal expansion are often only 10% or less of the total bolt load, which is of the order of typical pretension uncertainty. We study here a single, unconstrained joint consisting of a steel bolt-nut-washer set clamping a cylindrical piece of aluminum (Figure 2). For this configuration, the washers have the same outer diameter as the bolt head and nut; in designs where this is not the case, the washer area should be based on a frustum. Also, the bolt shank completely fills the hole. Normally, sufficient clearance is designed between the shank and hole such that positive clearance is maintained throughout the service life; therefore, we do not model contact between the shank and hole. In the few cases where the shank expands more than the hole, the surfaces simply interpenetrate without developing any contact stress. Shank-hole interference in a poorly designed or manufactured joint can significantly affect the bolt force.

B. Nomenclature

Most symbols used in this document are defined in Table 1. In general, material and geometric properties are subscripted f for flange, b for bolt and nut, w for washer and m for the clamped members (flanges and washers) as a set.

C. Flange Effective Area and Bolt Effective Length

Calculating the change in bolt force in a heated bolt-sleeve assembly is a classic problem in introductory mechanics of materials. The textbook problem is 1-D: the entire cross-sectional area of the sleeve evenly resists the thermal strain in the bolt; there is no external loading tending to bend the bolt or displace the compressed members; and the bolt is not compressing two or more contacting members but rather a single, continuous sleeve. Hand calculations of thermal stress are invariably based on this 1-D idealization of the joint, with prying effects applied as a modification.
Figure 2. Joint under study, shown disassembled on left for clarity. We varied the design by changing the thickness and diameter of flange, the bolt length, and the flange material. For this example, $r = 0.75$ and $s = 2.67$.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$A$</td>
<td>cross-sectional area</td>
<td>$L_w$</td>
<td>thickness of each washer</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>coefficient of thermal expansion</td>
<td>$P_{\text{bolt}}$</td>
<td>bolt force</td>
</tr>
<tr>
<td>$d$</td>
<td>diameter</td>
<td>$P_{\text{ext}}$</td>
<td>external force</td>
</tr>
<tr>
<td>$d_{\text{avail}}$</td>
<td>diameter of the largest circle centered on the bolt that can be inscribed on the flange face</td>
<td>$P_{\text{ini}}$</td>
<td>bolt initial pretension</td>
</tr>
<tr>
<td>$d_{\text{lim}}$</td>
<td>maximum diameter of compression frustum</td>
<td>$P_{\text{sep}}$</td>
<td>separation load</td>
</tr>
<tr>
<td>$\delta$</td>
<td>change in length along bolt axis</td>
<td>$\Delta P$</td>
<td>change in pretension due to thermal expansion</td>
</tr>
<tr>
<td>$e_b$</td>
<td>hole edge distance</td>
<td>$r$</td>
<td>ratio of bolt diameter to flange thickness</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus</td>
<td>$s$</td>
<td>ratio of flange diameter to washer diameter</td>
</tr>
<tr>
<td>$f_{\text{pry}}$</td>
<td>prying factor</td>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$g$</td>
<td>grip</td>
<td>$\theta$</td>
<td>compression frustum half-angle</td>
</tr>
<tr>
<td>$k$</td>
<td>axial stiffness</td>
<td>$w$</td>
<td>ESA blind hole correction</td>
</tr>
<tr>
<td>$L_{tf}$</td>
<td>total thickness of clamped flange or flanges</td>
<td></td>
<td></td>
</tr>
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</table>
Most studies and hand analyses start from an equation of the form

$$P_{\text{bolt}} = \Phi P_{\text{ext}} + P_{\text{ini}} + \Delta P,$$

(1)

where $\Phi$ is a joint stiffness ratio whose form depends on the arrangement of the bolt, washers and other clamped members, and on where the external load is assumed to enter the joint. To use Equation (1) to calculate the bolt force, values for the various $E$, $A$ and $L$ quantities, as well as the preload, must be established. But let us back up and examine the assumptions that underlie this equation.

Equation (1) treats the members as a network of preloaded springs whose length changes must be compatible. We refer to the distance over which this length compatibility is enforced as the joint gauge length (JGL). Thermal expansion will tend to change the pretension by an amount $\Delta P$ proportional to the temperature rise $\Delta T$ relative to the installation temperature. By static equilibrium, the force change $\Delta P$ in the bolt is equal and opposite to the force change $-\Delta P$ in the washers and flange. Assuming 1-D, linear elastic behavior, the bolt will therefore elongate by an amount

$$\delta_b = \alpha_b L_b \Delta T + \frac{L_b}{E_b A_b} \Delta P,$$

(2)

and each washer and the flanges will elongate by

$$\delta_w = \alpha_w L_w \Delta T - \frac{L_w}{E_w A_w} \Delta P, \quad \delta_f = \alpha_f L_f \Delta T - \frac{L_f}{E_f A_f} \Delta P,$$

(3)

respectively. The “bolt” elongation includes relative axial motion between the nut face and the bolt, as well as the deformation of the bolt itself. The relative motion arises because of thread slippage and the elasticity of the nut. Therefore, the threads and nut properties will affect the “bolt” stiffness.

The elongation of the bolt over the JGL must be compatible with the elongation of the flange-washers stack, so that

$$\delta_b = (\alpha_f L_f + 2\alpha_w L_w) \Delta T - \left( \frac{L_f}{E_f A_f} + 2 \frac{L_w}{E_w A_w} \right) \Delta P.$$

(4)

The use of Equation (1) demands that both Equation (2) (the bolt elongation) and Equation (4) (the stack elongation) produce the same value for $\delta_b$. Otherwise, the assumed elongation compatibility does not exist. Furthermore, $\delta_b$ has to equal the value measured or recovered from simulation. For design purposes, $\delta_b$ can often be ignored, but this is not always the case, and $\delta_b$ is frequently considered in experiments or stiffness studies such as the present one.

Because neither the fasteners nor the clamped members behave exactly as uniaxial, linear elastic bodies, neither Equation (2) nor Equation (4) will predict $\delta_b$ accurately if the actual $E$, $A$, $L$ and $\alpha$ values are used as inputs. The only way these equations can both match the measured value of $\delta_b$ is to choose at least one parameter in each equation as an effective value to be adjusted in order to obtain the correct value of $\delta_b$. Conventionally, and in the present paper, these are taken to be the flange area $A_f$ and the bolt length $L_b$. If we do not permit ourselves to adjust $L_b$, Equation (2) will not predict a value of $\delta_b$ compatible with the assumed 1-D model.

It is tempting to think of these effective values as the portion of the joint carrying significant load, but in reality, neither the area nor the length of any component is fully effective, even though some are assumed so. The effective parameters $A_f$ and $L_b$ are really just adjustment factors, subject to certain restrictions imposed by the 1-D model, to compensate for all of the approximations in the hand formulas, including neglect of yielding.
Rearranging Equations (2) and (4) yields equations we can use to extract \( L_b \) and \( A_f \) from measured or simulated values of \( \delta_b \) and \( \Delta P \):

\[
\tilde{L}_b = \frac{L_b}{g} = \frac{E_b A_b \delta_b}{g(\Delta P + E_b A_b \alpha_b \Delta T)}
\]

\[
\tilde{A}_f = \frac{A_f}{L_f^2} = \frac{E_w A_w \Delta P}{E_l L_f^2 (E_w A_w (L_f \alpha_f \Delta T - \delta_b) + 2 L_w (E_w A_w \alpha_w \Delta T - \Delta P))}
\]

where we have nondimensionalized \( L_b \) and \( A_f \).

We must now precisely define the JGL. Before the fastener is tightened or heated, the JGL, over which \( \delta_b \) is measured, is clearly equal to the grip. But once deformation begins, the bolt head, washers and nut undergo “dishing” such that the grip becomes shorter at the centerline than at the exposed outer radius; see Figure 3. Equations (2) and (4) apply to any elongations measured between Surface A and Surface B in Figure 3, but we must choose a definite radial location for the JGL. Any points on Surfaces A and B could be chosen, but the only points reasonably accessible to a measuring device are Points X and Y (Figure 3). Therefore, we define the JGL, over which \( \delta_b \) is measured, as the radial distance between Points X and Y. A different choice of radial location would lead to a different value of \( \delta_b \), and, therefore, different values of \( L_b \) and \( A_f \) from Equations (5) and (6). Those different values would be valid provided they both were calculated from the same value of \( \delta_b \), measured on some consistent radius between Surfaces A and B, but they would be invalid if \( \delta_b \) were measured at inconsistent radii or not between Surfaces A and B.

Mechanical design textbooks tend to ignore these details. One widely cited study\(^1\) measures elongation at a centerline node on the top of the fastener, but avoids error because it assumes a fastener that cannot deform or tilt. That method could not be used with an elastic fastener. The commonest approach is to extract \( A_f \) from the bolt force and the radially averaged elongation over the grip but to set \( L_b \) equal to the grip. This will upset the assumed elongation compatibility and lead to an error in bolt force predictions. Also, measuring displacement anywhere on the contact surface but at the exposed edge can only be done with difficulty, if at all. One study\(^9\) recognizes the importance of adjusting \( L_b \) to fit the data, but calculates stiffness from the change in elastic strain energy in the entire part. That can only succeed in a simplified, elastic analysis without friction or thread slippage, and does not tell the experimenter where to measure elongations. We will show later that a valid and consistent definition of bolt elongation is required not only for mathematical consistency but also for quantitative accuracy.

The stress is nearly uniform over the length of bolt that lies inside the bolt hole, but the tension drops to zero at the top of the head and the end of the shank. By claiming that the bolt tension increase \( \Delta P \) is equal to the washer compression increase, we are implicitly defining \( P_{\text{bolt}} \) as the normal contact force between the bolt head and washer. That force equals the tension across any bolt cross section between Surfaces A and B. It is difficult to measure \( P_{\text{bolt}} \) experimentally. Conventional methods such as ultrasound actually measure the bolt elongation, and then determine \( P_{\text{bolt}} \) from an assumed relation between elongation and force. But we need independent measurements of bolt force and elongation in order to extract independent values of \( \delta_b \) and \( \Delta P \). Various internally instrumented bolts have been devised,\(^{10}\) but a theoretical correction would have to be made for the difference in behavior between an instrumented test bolt and an everyday, solid bolt. In the FE model, of course, \( P_{\text{bolt}} \) can be recovered at any section. We recover it midway down the shaft.

A review of the literature\(^{11,12,13,14}\) revealed many different formulas for the effective flange area \( A_f \) and effective bolt length \( L_b \), and also showed that the flange thickness \( L_f \) and bolt area \( A_b \) are assumed to be fully effective; that is, their actual geometric values are used in the formulas, not some effective value. Also, since the washers are thin relative to the bolt head and flanges, it may be assumed that they are fully effective in compression provided they are not much larger in radius than the bolt head. In other words, the various quantities are determined as listed in Table 2.
Figure 3. Definition of joint grip length (JGL) over which the bolt elongation is measured. Note the “dishing” under load.

Table 2. Assumptions common to bolted joint formulas in the literature

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Calculated from</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_f$</td>
<td>Actual thickness (see Figure 5)</td>
</tr>
<tr>
<td>$A_b$</td>
<td>Shank area (may be corrected for variable area or threaded portion in grip)</td>
</tr>
<tr>
<td>$L_w$</td>
<td>Actual thickness (see Figure 5)</td>
</tr>
<tr>
<td>$A_w$</td>
<td>Actual area (overhang can be corrected for by using a frustum)</td>
</tr>
<tr>
<td>$L_b$</td>
<td>Approximate formulas based on geometry</td>
</tr>
<tr>
<td>$A_f$</td>
<td>Approximate formulas based on geometry</td>
</tr>
</tbody>
</table>
The geometric formulas for effective joint area and bolt length found in the literature are generally functions solely of the ratio of bolt diameter to flange thickness, \( r \equiv \frac{d_b}{L_f} \). However, one method\(^{14}\) also includes dependence on the ratio of available flange diameter to underhead bearing diameter (here equal to \( d_w \)): \( s \equiv \frac{d_{\text{avail}}}{d_w} \).

D. Finite Element Model Development

We created Abaqus finite element models to provide numerical predictions of elongation and bolt force for a range of flange thicknesses, diameters and materials. The models used fully integrated first-order quadrilaterals; we found that the default reduced-integration elements suffered from hourglassing and produced unphysical displacements in regions of high strain. We confirmed that the results did not change appreciably when second-order elements were substituted, indicating a sufficiently refined mesh. Nonlinear geometry was activated in all analyses. We modeled the bolt, washer and flange as separate bodies in hard contact, except that the bolt shank and hole are allowed unlimited interpenetration, to simulate the usual clearance; the actual interpenetration is a small fraction of the bolt diameter. The threads were modeled with the standard UNF thread geometry (24 threads per inch for a 3/8-inch bolt).

Lehnhoff and Bunyard\(^5\) have shown that when the threads are modeled in detail, there can be a significant decrease in bolt stiffness as compared to a simplified model with no threads (nut and bolt effectively a continuous body). We confirmed this with a study to be described below. We observed that most of the additional compliance is due to the nut threads slipping relative to the bolt threads, rather than bending of the threads. Therefore, the FE mesh in the three-dimensional model to appear later captures the effect adequately with only one element face across each thread face. The only obvious geometric difference between our models and a real fastener joint is that our threads are idealized as individual circular grooves instead of a single helical groove. We do not believe this difference has a significant effect on the behavior of interest in this study. Fukuoka and Nomura\(^{15}\) have modeled threads with their actual, helical geometry and were able to predict detailed thread stresses, but they did not investigate the effect on overall joint behavior.

The as-manufactured flatness of the mating flanges may have a significant effect on joint behavior. A wavy flange may introduce initial local gaps similar in size to the deformations that arise due to external loads. However, this factor is outside the scope of the present paper. Our models, as well as the hand calculation methods we consider, assume the flange faces to initially be absolutely flat.

In this section, we have chosen to model a bolt clamping a single flange rather than a stack of one or more flanges. A single flange results in a slightly higher flange effective area than a stack of two separate half-thickness flanges. However, the difference is no more significant than the influence of flange diameter, flange material, or the other design differences we investigated, and would not change the overall conclusions, an observation also made by Sethuraman and Kumar.\(^8\) \textit{This observation only holds true for the special circumstances of no external load or constraint.} When we later extend the results of this section to an externally loaded flange joint, the flanges will not remain in continuous contact, resulting in nonlinear clamped mem-

![Figure 4. Linear elastic and elastic-plastic stress-strain curves used in the bolt/nut models](image-url)
ber (and overall joint) stiffness. This difference is one reason we must evaluate our initial, idealized results in a flange joint before declaring them to be valid for that type of joint.

Table 3 lists the material properties. We consider two constitutive models for the fasteners: a linear elastic model and the simple bilinear elastic-plastic law shown in Figure 4. The bolt was assumed to be a high-strength, SAE Grade 8, 120 ksi proof bolt, with yield at 130 ksi. The elastic-plastic curve assumes a linear progression from yield to an ultimate strength of 150 ksi at 12% total elongation. Yielding is governed by an isotropic, rate-independent Mises yield surface. The same material was used for the nut.

<table>
<thead>
<tr>
<th>Component</th>
<th>Material</th>
<th>(E) (psi)</th>
<th>(\alpha) (/°F)</th>
<th>(\nu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum flange</td>
<td>Aluminum</td>
<td>10.0 (\times) 10^6</td>
<td>12.6 (\times) 10^{-6}</td>
<td>0.33</td>
</tr>
<tr>
<td>Titanium flange</td>
<td>Ti-6Al-4V</td>
<td>16.9 (\times) 10^6</td>
<td>4.7 (\times) 10^{-6}</td>
<td>0.31</td>
</tr>
<tr>
<td>Bolt, washers, nut</td>
<td>High-strength steel</td>
<td>30.0 (\times) 10^6</td>
<td>9.1 (\times) 10^{-6}</td>
<td>0.30</td>
</tr>
<tr>
<td>Elastic-plastic bolt and nut</td>
<td>Elastic-plastic law in Figure 4, otherwise same as above</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figures 5 and 6 show results for a 3/8-inch steel bolt clamping a 0.5-in-thick aluminum flange stack and two flat steel washers, each 0.078 in thick. Figure 6 shows the predicted increase in bolt force due to a temperature increase of \(\Delta T = 230\) °F, for a range of initial pretension. Predictions were made using two linear elastic models: first, a model in which the nut and bolt were modeled as a single, continuous body (no thread modeling), and second, a model with a detailed representation of the nut and bolt threads in frictional contact. Results from a third model, with detailed threads and using the elastic-plastic fastener material (Figure 4), are shown.

The results show a significant difference in the predicted bolt force change depending on whether the threads are modeled in detail. The simplified model with no threads overpredicts the bolt force change by 21% to 47% over the pretension range considered, and shows that the force change slightly increases with increasing initial pretension, whereas the threaded model shows that the force change slightly decreases with increasing initial pretension. Therefore, modeling the threads is a significant improvement in fidelity and will be implemented in the remainder of this paper.

The effect of thread plasticity is less significant. Up to an initial pretension level of almost 75% of proof, the threads do not yield at all. At higher pretension levels, the threads begin to yield such that the flange can expand without loading the bolt quite as much as it would had the threads remained elastic. The difference between the linear elastic and elastic-plastic models amounts to 10-15% as bolt tension approaches the bolt’s proof strength. The proof tension for the 120 ksi bolt, using the tensile stress area, is 10500 lb. Such a bolt would ordinarily be tightened to 75% of its proof load, or 7900 lb. These load levels are shown for reference on Figure 6. Although fastener plasticity only affects the thermal load results at high preloads, and then only slightly, we model the fasteners as elastic-plastic in the remainder of the paper because this allows identification of gross joint failure under external loading.

**E. FE Parameter Studies**

We now show results from the FE runs to illustrate the effects of friction, flange material, flange dimensions, and whether the joint is heated or cooled. With one exception, the hand methods only depend on \(r\), so the differences other than in \(r\) may be regarded as uncontrolled variables when using the hand methods. A useful hand formula should provide adequate accuracy regardless of these differences.
Figure 5. Section view of axisymmetric FE model of bolted joint with dimensions (left) and typical Mises stress distribution plotted on deformed geometry (displacements scaled up 20×.).

Figure 6. Predictions of bolt load change due to $\Delta T = 230^\circ$F, with three different FE models and over a range of initial pretension levels.
We created models with eight different combinations of flange thickness and radius, and ran those sets for an aluminum flange and a titanium flange under both heating and cooling conditions. Changes in flange thickness correspond to different values of \( r \), while changes in flange radius correspond to different values of \( s \). The bolt diameter was kept the same (3/8 inch) in all models; only the length was changed, as necessary to accommodate the various flange thicknesses. In all runs, an initial pretension of 7900 lb (75% of proof) was applied by shortening the bolt at its midsection. Figure 7 depicts the models in order to show the appearance of the different designs. We used Equations (2) and (4) to extract \( \tilde{A}_f \) and \( \tilde{L}_b \) from the FE-calculated \( \delta_b \) and \( \Delta P \).

\[ s = 2.67 \]
\[ s = 8.00 \]
\[ r = 0.250 \quad r = 0.444 \quad r = 0.625 \quad r = 0.750 \]

Figure 7. The eight combinations of flange thickness and radius used in the FE simulations

1. Friction

To investigate the effect of friction, which is often not known accurately, we varied the coefficient of friction on all contact pairs from 0 to 0.5. This had less than a 1% effect on the extracted values of \( \tilde{A}_f \) and \( \tilde{L}_b \); see Figure 8. For the remainder of this section, a friction coefficient of 0.15 is used.

\[ \tilde{A}_f \]
\[ \tilde{L}_b \]

Figure 8. Comparison of FE-derived values of \( A_f \) (left) and \( L_b \) (right) for high friction \((\mu = 0.5)\) and zero friction
2. Aluminum flange versus titanium flange

The flange effective area extracted from the FE results is slightly smaller for a titanium flange versus an aluminum flange, and the bolt effective length is about 20% smaller for titanium versus aluminum (Figure 9). The comparison here is between an aluminum flange that is heated, and a titanium flange that is cooled, such the both runs produce an increase in bolt force. However, looking ahead to Figures 11 and 12, it can be seen that the bolt effective length difference is primarily a function of the flange modulus and not whether the bolt force increases or decreases.

Aluminum differs from titanium in both modulus and thermal expansion coefficient. To investigate which parameter was associated with the difference in \( \tilde{L}_b \), we ran a study varying \( \alpha_f \) from the titanium value (4.7 \( \times \) 10\(^{-6}\) \(/{ }^\circ\)F) to the aluminum value (12.6 \( \times \) 10\(^{-6}\) \(/{ }^\circ\)F). We did this for two different values of \( E_f \): the aluminum value and the titanium value. The results (Figure 10) showed that values of \( \tilde{L}_b \) were strongly affected by \( \alpha_f \), but practically overlaid each other for the two values of \( E_f \).

It should be kept in mind that the data points in Figure 10 represent combinations of \( \alpha \) and \( E \) that are not necessarily representative of any real material. An interesting observation in the course of these runs was that changing both \( E \) and \( \alpha \) from the aluminum to the titanium values did not quite reproduce results we had previously obtained for titanium. The difference was that Poisson’s ratio \( \nu \) had been left at the value for aluminum (0.33). Poisson’s ratio has a small but noticeable effect on joint behavior in and of itself, as has been observed by Pedersen and Pedersen. This effect is accounted for in our effective values for \( A_f \) and \( L_b \).

3. Heating versus cooling

For the aluminum flange, the effective flange area is generally a few percent smaller, and the effective bolt length a few percent larger, for heating versus cooling (Figure 11). For the titanium flange, the opposite is true (Figure 12). However, in both cases, the effect is relatively minor.
Figure 11. Comparison of FE-derived values of $A_f$ (left) and $L_b$ (right) for heating versus cooling, for aluminum flange

Figure 12. Comparison of FE-derived values of $A_f$ (left) and $L_b$ (right) for heating versus cooling, for titanium flange
4. Ratio of flange available diameter to bolt diameter

The flange effective area increases when the flange radius increases (Figure 13). The effect is small compared to the effect of changing the flange material. The bolt length is unaffected by the flange diameter. The direction and magnitude of the effect is the same for both an aluminum and a titanium flange, so only the results for an aluminum flange are shown.

Figure 13. Comparison of FE-derived values of \( A_f \) (left) and \( L_b \) (right) for small-diameter aluminum flange and large-diameter aluminum flange

F. Hand-Calculation Formulas

We reviewed the literature to find formulas for the flange effective area and bolt effective length. All of the methods except for Method C are solely a function of the ratio of bolt diameter to flange thickness, which we denote as \( r \). Method C also depends on the ratio \( s \) of flange available diameter \( d_{avail} \) to washer diameter \( d_w \).

It is important to note that the textbooks and papers in which formulas are given all recognize that their equations are approximate and suggest a more detailed experimental or numerical study if the application is critical. The present investigation is in the spirit of a more detailed study of a specific, critical application. Identification of shortcomings in the accuracy of the hand-calculation formulas for this application are not intended to be general criticisms of the formulas, and they may be well-suited to situations in which accuracy can be traded for ease of analysis.

Method A

Method A is the method appearing in the mechanical design textbook of Shigley:  

\[
\hat{k} = \frac{\pi E_f d_b \tan \theta}{\ln \left( \frac{(L_f \tan \theta + d_w - d_b)(d_w + d_b)}{(L_f \tan \theta + d_w + d_b)(d_w - d_b)} \right)}
\]

for the stiffness of each of the clamped flanges, where \( 25^\circ \leq \theta \leq 33^\circ \) is the half-angle of an imaginary conical frustum of effective compression within each flange. Shigley recommends a half-angle of \( 30^\circ \) be assumed for most joint designs unless there is insufficient flange area to contain the resulting compression frustum. Another investigator recommends a half-angle of \( 45^\circ \), which is outside the range recommended by Shigley but will be considered here as a possible alternative. A stack of two identical flanges, of total thickness \( L_f \), will have a stiffness equal to \( \hat{k}/2 \). Therefore,

\[
\frac{k}{2} = \frac{E_f A_f}{L_f},
\]

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American Institute of Aeronautics and Astronautics
so

\[ A_f = \frac{k L_f}{2 E_f}. \]  

(9)

Combining Equations (7) and (9) and substituting \( d_w = 1.5 d_b \),

\[ A_f = \frac{\pi L f d_b \tan \theta}{2 \ln \left( \frac{L f \tan \theta + 0.5 d_b}{L f \tan \theta + 2.5 d_b} \right)} \]  

(10)

If the washer overhangs (that is, if \( d_w > 1.5 d_b \)), the more general formula provided by Equation (7) should be used. Writing the result in terms of the ratio \( r \equiv d_b / L_f \) and nondimensionalizing,

\[ \tilde{A}_f \equiv \frac{A_f}{L_f^2} = \frac{\pi r \tan \theta}{2 \ln \left( 5 \frac{\tan \theta + 0.5 r}{\tan \theta + 2.5 r} \right)} \]  

(11)

Method A uses a formula for bolt stiffness that is the series stiffness of the threaded and unthreaded portions of the part of the bolt lying within the grip. There is no correction for the compliance of the bolt head or other fastener parts lying outside the grip. Since the designs we study here have no threaded length within the grip, the dimensionless effective bolt length is simply

\[ \tilde{L}_b \equiv L_b / g = 1. \]  

(12)

**Method B**

Method B is that of Juvinall,\(^{12}\) and is also based on the conical compression frustum idea. However, Method B involves an approximation to the volume of the frustum and assumes a half-angle of 30°. It also includes the assumption \( d_w = 1.5 d_b \), which is consistent with the design addressed in this section. The formula is

\[ A_f = \frac{\pi}{4} \left[ \left( \frac{3 d_b + L_f \tan 30^\circ}{2} \right)^2 - d_b^2 \right]. \]  

(13)

Substituting \( d_b = r L_f \), using \( \tilde{A}_f \equiv A_f / L_f^2 \), and simplifying,

\[ \tilde{A}_f = \frac{\pi}{4} \left( \frac{5}{4} r^2 + \frac{\sqrt{3}}{2} r + \frac{1}{12} \right) \]  

(14)

Method B, like Method A, also presents a formula for bolt stiffness that is simply the series stiffness of the threaded and unthreaded portions of the part of the bolt lying within the grip. Therefore, for the problem considered here in which the threads are not explicitly modeled, the dimensionless effective bolt length is again \( \tilde{L}_b = 1 \).

**Method C**

Method C, taken from a draft document of the European Space Agency,\(^{14}\) is considerably more complicated than Methods A and B. It accounts for the fact that there may not be enough flange area for the compression frustum to develop fully. It also separates the compressed material into a sleeve (directly beneath the bolt head or washer) and a cone (which is truncated if the flange does not extend far enough to allow it to fully develop).

To check whether the frustum has room to fully develop, the maximum diameter of the frustum,

\[ d_{\text{lim}} = d_w + w L_f \tan \theta \]  

(15)
is compared to the available flange diameter \( d_{\text{avail}} \). For a nut joint, \( w = 1 \). In the case \( d_{\text{avail}} \geq d_{\text{lim}} \), the frustum has room to fully develop, and the flange stiffness is given by a formula identical to that of Method A except that the half-angle is a function of the joint geometry instead of a constant:

\[
\tilde{A}_f = \frac{\pi r \tan \theta}{2 \ln \left( \frac{5 \tan \theta + 0.5r}{\tan \theta + 2.5r} \right)}, \quad \tan \theta = 0.265 - 0.032 \ln r + 0.153 \ln s. \tag{16}
\]

In the above equation, the formula for \( \tan \theta \) is a function of \( r \equiv d_b/L_f \) and \( s \equiv d_{\text{avail}}/d_w \).

If the frustum does not have room to fully develop (i.e. \( d_{\text{avail}} < d_{\text{lim}} \)), a modified flange stiffness formula is used, given as follows:

\[
\tilde{A}_f = \frac{\pi r \tan \theta}{2 \ln \left( \frac{5s-1}{s+1} \right) + \frac{4(\tan \theta - rs + 1.5r)}{r(s^2-1)}}. \tag{17}
\]

Note that when the frustum becomes limited by the available flange area (\( d_{\text{avail}} = d_{\text{lim}} \)),

\[
s = 1.5 + \frac{\tan \theta}{r}, \tag{18}
\]

and the correction term (the second term in the denominator of Equation (17)) becomes zero. Also note that when there is no flange material except that directly under the washer, \( s = 1 \) and Equation (17) blows up. In this case, all of the available flange material is under compression, so the flange effective area is equal to its actual area.

Method C, unlike Methods A and B, corrects the bolt effective length for the compliance of the parts of the fasteners outside the grip. For the design under consideration here, the effective length of the bolt is the length of the gripped shank plus “substitution lengths” of 0.4\( d_b \) for the head, the nut, and the portion of the shank inside the nut. Thus,

\[
L_b = g + 0.4d_b + 0.4d_b + 0.4d_b \left( \frac{d_b}{d_{\text{min}}} \right)^2, \tag{19}
\]

so

\[
\tilde{L}_b = 1 + \frac{0.8d_b + 0.4d_b \left( \frac{d_b}{d_{\text{min}}} \right)^2}{g}. \tag{20}
\]

G. Comparison of Hand Calculations to FE Results

In this section, we compare Methods A, B and C to the detailed axisymmetric FE model, in terms of the flange effective area, bolt effective length, and bolt force change. The same FE results presented in the preceding section are compared here to the hand formulas (Figure 14). All of the FE results, including variations in flange material, friction, temperature are plotted on this one graph, because a valid hand formula should be able to match the data well under all these conditions. The only data handled separately are those corresponding to different flange diameters \( s \).

The left-hand plot in Figure 14 shows that all of the hand formulas capture the trend of increasing \( A_f \) with increasing \( r \) well. Method A gives good agreement provided the lower-bound cone angle of 25° is used. Method C produces nearly identical results, though it uses a variable cone angle. The hand methods result in equal or larger estimates of \( A_f \) as compared to our FE results. But the more important comparison is the bolt force predictions, which will also involve \( L_b \).

The right-hand plot shows that, as seen in the preceding section, the flange material has a strong effect on the bolt effective length. Methods A and B, which assume that the bolt effective
length is equal to the grip length, split the two sets of data. The unusual bolt length corrections in Method C tend to greatly overestimate the bolt effective length, especially when the bolt is short in relation to the head and nut lengths (high values of $r$).

In Figure 15, we take a closer look at the effect of flange diameter, as captured by the parameter $s$. Method C shows the correct dependence of flange effective area on $s$, and does a good job of estimating the difference between $s = 2.67$ and $s = 8.00$. However, the effect is no more significant than that of other parameters such as flange modulus which do not appear in Method C.

Figure 14. Comparison of FE-derived results to hand methods for flange effective area (left) and bolt effective length (right). The plot at right uses different markers for aluminum and titanium flanges, to show the different trends. For all these calculations, $s = 2.67$.

Figure 15. Comparison of Method C predictions to FE results for two different values of dimensionless flange radius $s$

Summarizing, Methods A (with $\theta = 25^\circ$) and C produce the best agreement with the FE-derived flange effective area. Method C has the ability to capture variations in $s$, but this perhaps is not necessary considering that other equally significant effects are not captured. None of the methods was in good agreement with the FE-derived bolt effective length, which varied significantly.
depending on the flange material. The right-hand plot of Figure 14 suggests that choosing separate values of $\tilde{L}_b$ for aluminum and titanium flanges might result in significantly more accurate hand calculations.

As a final check, we compare the Method A hand-calculated bolt force to the bolt force from the FE analysis. Equations (2) and (4) may be combined to yield

$$\Delta P = \frac{(\alpha_f L_f + 2\alpha_w L_w - \alpha_b L_b) \Delta T}{\frac{L_b}{E_b A_b} + \frac{L_f}{E_f A_f} + 2 \frac{L_w}{E_w A_w}},$$

(21)

giving the equation for bolt force when $A_f$ and $L_b$ are provided by the hand methods. We then used Method A with $\theta = 25^\circ$ to calculate $\tilde{A}_f$; Method C gives nearly identical results but is more complicated. For $\tilde{L}_b$, we used dimensionless values of 1.08 for the aluminum flange and 0.83 for the titanium flange; these gave very good fits to the FE results (Figure 16). For comparison, we also show the results from Method A with $\tilde{L}_b = 1.00$; the resulting overestimates in bolt force change are substantial (30% to 90%). An overestimate of the bolt force can be desirable or undesirable depending on the situation. Based on the results of this study, in the remainder of this paper we will calculate $\tilde{A}_f$ by Method A with $\theta = 25^\circ$, and use $\tilde{L}_b = 1.08$ for the aluminum flange.

Figure 16. Thermally induced change in bolt force as calculated by hand and FE for aluminum flange (top) and titanium flange (bottom)
We have stated that the JGL must be defined in a consistent and valid way. We show in Table 4 the practical importance of doing this. First, we recover elongation from the aluminum-flange FE model with \( r = 0.750 \) and \( s = 2.67 \), using different definitions of the JGL. We then compare the bolt force (Equation (21)), bolt elongation (Equation (2)) and stack elongation (Equation (4)) predicted by various possible combinations of \( \tilde{A}_f \) and \( \tilde{L}_b \) that can be derived from the data. The first three methods use consistent and valid definitions of the JGL and produce valid results. The fourth uses inconsistent radii and gives incompatible results. The fifth uses the radially averaged elongation but sets \( L_b \) equal to the grip (the value is boxed in the table to emphasize this); it gives incompatible elongations and an erroneous bolt force. The errors are small in this example, but they are unnecessary. The sixth uses the entire centerline bolt length as the JGL and gives consistent answers for bolt force and elongation, because the combined bolt and member stiffness is correct. But the value of \( \tilde{A}_f \) obtained by this sixth method is actually negative, so the split between bolt and member stiffness is badly in error. The results emphasize the importance of specifying a consistent and valid measurement of bolt elongation when extracting the effective values from experiments or simulations.

Note that choosing \( \tilde{L}_b = 1.08 \), which produces the best overall fit over a range of joint geometries, does introduce an inconsistency and will result in an inexact prediction of bolt elongation. If a very accurate and exactly consistent prediction of bolt elongation is desired, \( \tilde{L}_b \) should be adjusted separately for each geometry of interest.

Table 4. Sample calculations showing the errors resulting from inconsistent or invalid definition of the JGL. Displacements are in mils and force is in lb.

<table>
<thead>
<tr>
<th>JGL</th>
<th>( \delta_b )</th>
<th>( \tilde{A}_f )</th>
<th>( \tilde{L}_b )</th>
<th>( \Delta P )</th>
<th>Eqn (2)</th>
<th>Eqn (4)</th>
<th>Valid?</th>
</tr>
</thead>
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<tr>
<td>Between Points X &amp; Y</td>
<td>1.64</td>
<td>0.917</td>
<td>1.108</td>
<td>536</td>
<td>1.64</td>
<td>1.64</td>
<td>Y</td>
</tr>
<tr>
<td>Between Surfaces A &amp; B,</td>
<td>1.52</td>
<td>0.460</td>
<td>1.029</td>
<td>536</td>
<td>1.52</td>
<td>1.52</td>
<td>Y</td>
</tr>
<tr>
<td>radially averaged</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Between Surfaces A &amp; B,</td>
<td>1.48</td>
<td>0.387</td>
<td>0.999</td>
<td>536</td>
<td>1.48</td>
<td>1.48</td>
<td>Y</td>
</tr>
<tr>
<td>on centerline</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Between Surfaces A &amp; B,</td>
<td>1.52</td>
<td>0.460</td>
<td>1.000</td>
<td>600</td>
<td>1.48</td>
<td>1.52</td>
<td>N</td>
</tr>
<tr>
<td>radially averaged,</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>bolt length not adjusted</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Between Surfaces A &amp; B,</td>
<td>NA</td>
<td>0.917</td>
<td>0.999</td>
<td>892</td>
<td>1.48</td>
<td>1.64</td>
<td>N</td>
</tr>
<tr>
<td>inconsistent radii</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entire bolt length,</td>
<td>3.48</td>
<td>-0.062</td>
<td>2.350</td>
<td>536</td>
<td>3.48</td>
<td>3.48</td>
<td>N</td>
</tr>
<tr>
<td>on centerline</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

III. Combined Loading in Flange Joint

A. Introduction

In the previous section, we evaluated various methods of calculating the effective bolt length and flange area based on geometry, by comparing them to detailed FE results for an axisymmetric joint with no external loading. We found that for joints with steel bolts and aluminum flanges, Method A with \( \theta = 25^\circ \) gave the best agreement for flange area, while a constant value of 1.08 gave the best agreement for dimensionless bolt effective length. In this section, we will use those methods to calculate member stiffnesses and again compare the results to detailed FE calculations. However,
here we will analyze an aluminum flange joint that is subjected to external loading as well as a large temperature change.

We carry over the 1-D, springs-in-parallel concept for developing hand formulas. However, the offset between the external load vector and the bolt centerline creates a prying action. From the insight gained from the axisymmetric model, we can say at the outset that prying may create analytical difficulties when the flanges “peel apart” and the bolt is simultaneously stretched and bent.

B. Hand Method

Returning to the 1-D model, when the external load is aligned with the bolt centerline, the bolt force is given by Equation (1). We modify this equation by inserting a prying factor $f_{\text{pry}}$, which is a multiplier on the bolt force due to the offset bolt centerline:

$$P_{\text{bolt}} = \Phi f_{\text{pry}} P_{\text{ext}} + P_{\text{ini}} + \Delta P$$

$$= \frac{k_b}{k_b + k_f + \frac{2k_b k_f}{k_w}} f_{\text{pry}} P_{\text{ext}} + P_{\text{ini}} + \Delta P.$$  \hspace{1cm} (22)

The formula here for the stiffness ratio $\Phi$ assumes that the external load enters the joint between the washers and flange. The use of a multiplicative factor to characterize the prying effect has been common for many years; one example is in Reference [10]. More sophisticated approaches to prying exist but are not assessed here. Some hand calculation methods also include a “loading plane factor,” which is meant to characterize the distribution of external load through the thickness of the flange; that factor is simply another multiplier on the external load and need not be broken out separately when a prying factor is used.

In an actual joint, and in our FE model, joint separation occurs gradually, as the flanges peel apart starting at the heel. However, in the 1-D model used for hand calculations, separation occurs all at once, when the external load increases to the point that the flanges lose contact around the fasteners, and only remain in contact at a “fulcrum” whose location remains to be defined (Figure 17). When the separation load has been reached, the bolt takes the entire external load, multiplied by the prying factor. The corresponding external load is

$$P_{\text{sep}} = \frac{(P_{\text{ini}} + \Delta P) \left( k_b + k_f + \frac{2k_b k_f}{k_w} \right)}{f_{\text{pry}} \left( k_f + \frac{2k_b k_f}{k_w} \right)}.$$  \hspace{1cm} (23)

The location of the fulcrum is yet another effective quantity that may be estimated from the joint geometry and used to calculate a bolt force that may be compared to test data or FE results. The fulcrum can be thought of as the line of action of the contact force between the flanges. In the late stages of joint failure, when the contact patch is receding from around the bolts, the fulcrum is clearly moving. However, we investigate here whether a fixed prying factor, can provide a useful estimate of bolt load over some range of external loading. Again, we assume perfect initial geometry, such that flange waviness or other imperfections have no influence.
Perhaps the simplest approach to estimating the prying factor is to assume that the contact force between the flanges, which is generated by compressing the flange over its effective area, acts only at the outer radius. In other words, the fulcrum is at the outer radius. Assuming that the external load acts halfway through the shell thickness, the prying factor is

\[ f_{\text{pry}} = \frac{t_f}{2} + \frac{w_b}{c_b}. \]  

(24)

Thus, the bolt force is

\[ P_{\text{bolt}} = \begin{cases} 
\frac{k_b}{k_b + k_f + \frac{2w_b}{c_w}} f_{\text{pry}} P_{\text{ext}} + P_{\text{ini}} + \Delta P, & P_{\text{ext}} < P_{\text{sep}} \\
f_{\text{pry}} P_{\text{ext}}, & P_{\text{ext}} \geq P_{\text{sep}} 
\end{cases} \]  

(25)

where the stiffnesses, as well as \( \Delta P \), are calculated using the method chosen in the previous section.

C. FE Model Development

Actual bolted joints are not as simple as those studied in the previous section. One common design is a flange connection between two cylindrical shells or housings (Figure 1). FE modeling of the entire joint is usually neither practical nor necessary, and would detract from the basic insights. It is sufficient to model a single fastener location, with boundary conditions carefully chosen to replicate the constraint existing in the actual joint, and to load the fastener with a force representing the local per-fastener force under whatever complex, nonuniform loading situation is actually of interest. Studies are also necessary to ensure that the FE mesh is fine enough and that contact conditions are adequately resolved. We describe those studies briefly in this subsection.

![Figure 18. Flange dimensions](image)

The axial external load is applied to a control node joined to the lower shell end by a multipoint constraint, and the upper shell end is fixed. We first confirmed that constraining the model tangentially as shown in Figure 19 gave results essentially identical to a costlier model with exact cyclic symmetry boundary conditions. Then, using meshes of different densities, we found that approximately 0.12 inch long, first-order incompatible-modes hexahedra were fine enough to resolve the stresses of interest away from the bolt hole, with locally increased density to 16 elements around the perimeter of the bolt hole and a finer mesh in the fasteners sufficient to resolve the threads.
However, that same mesh density resulted in puzzling jumps in the load-displacement characteristic. After study, we concluded that this spurious behavior was caused by contact discretization. We reached this conclusion by creating the simplest possible prying contact model (Figure 20), so as to eliminate other possible sources of discretization error. The same jumps in stiffness were seen, and these were mitigated, although not completely smoothed out, by using a finer mesh.

We found choosing an adequately long portion of shell, and choosing the constraint on the free end, to be the least straightforward aspect of modeling. Both of those factors influence the constraint imposed by the shells on the flanges, which in turn affects the joint behavior. The shell modeling we eventually chose is representative of some design situations but not others, and we can offer no general guidance except to say that when external loading is represented in a simplified fashion, as an axial force, enough shell must be modeled so that the end effects die out. In our model, we desired that enough shell be included to sufficiently reduce the difference in bolt force whether the cut shell ends were constrained from tilting or allowed to tilt. The other option is to model the shell all the way to a location where the load and constraint are known more definitely, but this could result in a very large and unwieldy model.

Figure 20. Simplified shell element contact model to show the effects of mesh density on joint stiffness during contact (left). Mesh refinement shows reduction in jumps (right).
Figure 19 shows the final flange joint model. The exact value of the shell radius is unimportant, as long as it is much larger than the fastener dimensions, as it is here. We defined hard contact with a small amount of penalty-formulation friction to improve convergence. As in the previous section, the bolt and nut are modeled with circular (not helical), but otherwise standard, UNF threads. Several levels of initial pretension are used, and the model is heated 150 °F, resulting typically in more than 500 lb of additional pretension. An external load is then applied at the lower shell end.

Table 5. Dimensions and material properties for the flange joint FE model

<table>
<thead>
<tr>
<th>Dimensions (inches)</th>
<th>Material Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_f$ 0.500</td>
<td>$\alpha_f$</td>
</tr>
<tr>
<td>$t_w$ 0.078</td>
<td>$12.6 \times 10^{-6}/^\circ F$</td>
</tr>
<tr>
<td>$d_w$ 0.688</td>
<td>$\alpha_w, \alpha_b$</td>
</tr>
<tr>
<td>$w_b$ 1.200</td>
<td>$9.1 \times 10^{-6}/^\circ F$</td>
</tr>
<tr>
<td>$h_h$ 0.375</td>
<td>$E_f$</td>
</tr>
<tr>
<td>$d_h$ 0.562</td>
<td>$10.0 \times 10^6$ psi</td>
</tr>
<tr>
<td></td>
<td>$E_b$</td>
</tr>
<tr>
<td></td>
<td>See Figure 4</td>
</tr>
<tr>
<td></td>
<td>$E_w$</td>
</tr>
<tr>
<td></td>
<td>$30.0 \times 10^6$ psi</td>
</tr>
</tbody>
</table>

$h_f = 2.000$ (“short”).

D. Results and Comparison to Hand Calculations

Figure 21 shows bolt force versus external load for an initial pretension of 7000 lb. We plot results for both the fixed and pinned shell end conditions to show that for the length of shell we modeled; the end tilt constraint has negligible effect. At an external load of about 6000 lb, gross yielding begins; we have made no attempt to accurately simulate the behavior after this point. A simulation with linearly elastic fasteners is included so that the onset of gross yielding in the elastic-plastic model can be identified by comparison.

Figure 21. Fastener tension vs. applied load for short and long shells using fixed and pinned constraints on the cut ends
Using methods developed in Section B, we calculated bolt force to compare to the FE results. With geometry defined in Table 5, we calculated $A_f$ by Method A (assuming $\theta = 25^\circ$) and assigned the dimensionless bolt effective length a constant value of 1.08. The effective area of the thin, overhanging washer was determined using a frustum. We then used Equation (25) to calculate the response to a 7000 lb preload, 150 °F temperature rise, and 9000 lb external load. The prying factor as calculated by Equation (24) is 2.3. The hand and FE results are compared in Figure 22.

Figure 22 shows that the hand method overpredicts fastener tension by 10-20% in this example. Note the all-at-once joint separation predicted by the hand method, as compared to the gradual separation displayed by the FE model. The contact patch between the flanges is approximately circular before external loading is applied. When loads are applied, the patch ovalizes, shifting the fulcrum away from the applied force. After separation, the hand methods predict that bolt force increases linearly, with the prying factor being the proportionality constant (here, 2.3) between the bolt force and the external load. But the FEM results for the latter part of the curve, when separation has become significant, are fit better by a slope of about 1.7.

We also show in Figure 22 the same simulation, but with bolt yielding suppressed. The purpose of this simulation is to show the load at which the slope of the bolt force curve becomes affected by fastener yielding. In this example, the bolt begins to take external load at about 2000 lb due to contact nonlinearity; this happens whether or not bolt yielding is modeled. Then, only in the elastic-plastic bolt model, bolt force undergoes a decrease in slope at an external load of about 7000 lb. This occurs as the bolt begins gross yielding.

Initial pretension has an effect on the degree of overprediction by the hand method. Figure 23 shows FE results compared to hand calculations for preloads of 3000, 7000 and 10500 lbf. Increasing preload tended to increase the amount of the overestimate. Again, an overestimate of bolt force can be desirable or undesirable depending on the situation.

For the flange joint, we can only assess the hand calculation of bolt and member stiffness in combination with the simple model for prying that we have adopted. We made no attempt to separate the two aspects, although this could be done by comparing a range of FE results to a more sophisticated hand analysis. Undoubtedly, the large changes in flange contact area affect the member stiffness, and these cannot be captured by the hand method, in which stiffness depends only on the initial geometry and is constant throughout the analysis.

![Figure 22. FEM bolt force prediction compared to hand-calculated values.](image)
Figure 23. Hand and both elastic and plastic FEM predictions for fastener tension for preloads of 3,000, 7,000 and 10,500 lbf with $f_{pry}=2.3$ (left). Redefining the prying factor as $f_{pry}=2.0$ produces a closer fit (right).

E. Joint Stiffness

Overall joint stiffness is often of interest, especially when dynamic loads must be considered. We define joint stiffness as the change in external load divided by the change in heel gap. This definition is useful when assessing coarse models in which the joint is represented only by a shared circle of nodes at the heel, or initially coincident heel nodes connected by a spring element. When shared nodes are used, the modeled joint stiffness is infinite, because there can be no displacement across the joint no matter the load. If a spring element is interposed at each bolt location, the modeled joint stiffness then corresponds to the definition just given. In multi-bolt flange designs where the heel gap varies significantly depending on proximity to a bolt, the gap must be averaged over one bolt sector.

When the heels are in contact, the joint is infinitely stiff in compression, because the heels cannot get any closer together. However, the heel nodes, while initially in contact, may separate due to warping of the flanges as the fastener is torqued down during installation. When this happens, the joint displays finite stiffness under low compressive loads, until compression is sufficient to bring the heels back into contact. The as-machined flatness of the flanges also affects this phenomenon. We do not show results for compressive loading in this paper.

Figure 24 shows that the joint stiffness decreases markedly and nonlinearly as soon as external tensile load is applied. Yielding plays no role until the joint is near failure; the initial, immediate softening is due to contact nonlinearity, as the flange surfaces separate. In this example, the stiffness decreases by over 50% at a tension load of 1000 lb, which is only about one-sixth of the external load capability of the joint. Such a large decrease in stiffness can significantly influence the dynamic response of the joint.

IV. Conclusions

We have assessed hand analysis methods for bolted joints by comparing predictions to detailed FE results. Study of a concentric, axisymmetric joint provided insight into basic joint mechanics and allowed development of a hand method capable of predicting preload change due to thermal expansion within 10% of the FE results. Some existing methods overpredicted the preload increase by as much as 90%.
Figure 24. Joint stiffness vs. applied load for six configurations. See the mesh density section for discussion of the irregular shape of the curves.

We then developed an FE model of a bolted flange joint in which offset between the external load and the bolt axis creates a prying effect. To apply the hand method developed in the first section, it was necessary to account for prying in the calculations of bolt force. The hand calculations thus incorporate simplified, linear representations of bolt and member stiffness, as well as prying. Based on the limited examples shown, the hand method tends to overestimate bolt force by 10-20% in the presence of prying, using the simplest way of calculating the prying factor.

References


