Proper Orthogonal Decomposition Analysis of Numerically Simulated Supersonic Jet Flow

J. Mignee\textsuperscript{1}, J. Kastner\textsuperscript{2}, D. Munday\textsuperscript{3}, N. Heeb\textsuperscript{4} and E. Gutmark\textsuperscript{5}

Gas Dynamics and Propulsion Laboratory, Aerospace Engineering Department, University of Cincinnati, Cincinnati, Ohio, 45220

J.Liu\textsuperscript{6} and K. Kailasanath\textsuperscript{7}

Naval Research Laboratory, Center for Reactive Flow and Dynamical Systems, Washington DC, 20375-5344

Proper Orthogonal Decomposition (POD) is performed on Large Eddy Simulation (LES) data and Particle Image Velocimetry (PIV) data from an underexpanded axisymmetric jet. PIV measures the streamwise and radial velocity components along a streamwise plane while LES provides data for the full domain and includes all flow variables (three velocity components, density, pressure, and temperature). A qualitative comparison between the POD modes from the LES and PIV data shows features similar in wavelength and shape particularly just downstream of the nozzle exit. An energy analysis shows that the biggest difference between the two cases was near the nozzle exit and believed to be the low turbulence level near the nozzle exit in LES data. At downstream positions, both cases have a similar distribution of POD modal energy. The analysis also shows that the large-scale flow features captured by the POD modes are strongly dependent on the domain size. Small domains are good for extracting the smaller-scale and low energy flow features near the nozzle exit. Further downstream the smaller domain spatially filters the largest flow features from the POD modes. The LES data is further post-processed to see if the streamwise and/or radial velocity can be used to estimate the tangential velocity, pressure, or density. It is found that the streamwise velocity can predict the pressure field, the radial velocity can predict the tangential velocity, and neither component does a good job at predicting the density. The prediction of flow variables is most likely due to the under-expanded nature of the jet and shows how PIV data can be used to further estimate other flow variables.

Nomenclature

\[(x,r,\theta)\] = Cylindrical coordinates

\[u, v, w\] = instantaneous respectively longitudinal, radial and tangential velocity component

\[U, V, W\] = mean respectively longitudinal, radial and tangential velocity component

\[u',v',w'\] = fluctuating part of respectively longitudinal, radial and tangential velocity component

\[N\] = number of grid points

\[\beta\] = orthonormal basis

\[\phi_i\] = basis vectors, eigenvectors

\[\lambda_i\] = \(i\)th POD eigenvalue

\(\langle,\rangle\) = inner product

\[\langle\rangle\] = average

\textsuperscript{1} Graduate Student, Aerospace Engineering Department, migneeje@mail.uc.edu, AIAA Student Member.

\textsuperscript{2} PhD, Aerospace Engineering Department, AIAA Member.

\textsuperscript{3} PhD Student, Aerospace Engineering Department, AIAA Member.

\textsuperscript{4} PhD Student, Aerospace Engineering Department, AIAA Member.

\textsuperscript{5} PhD, Distinguished Professor, Ohio Regents Eminent Scholar, Aerospace Engineering Department, AIAA Fellow.

\textsuperscript{6} Mechanical Engineer, AIAA Member.

\textsuperscript{7} Head, Center for Reactive Flow and Dynamical Systems, Fellow AIAA.
R(x,x') = two-point correlation tensor

\|\| = Norm

I. Introduction

Since the 1970’s coherent structures have been recognized as the physical back-bone of turbulent flow. The coherent structures possess a large portion of the fluctuating energy and therefore play a large role in many engineering problems such as noise, mixing, drag, etc. Thus, the physics of turbulent flows can be better understood by studying coherent structure dynamics. Unfortunately the nature of turbulent flows is such that as the Reynolds number increases a wide range of eddies and time scales coexist. Therefore this observation becomes challenging. In order to reduce the complexity of the flow and be able to capture the most energetic structures, Lumley introduced the Proper Orthogonal Decomposition (POD) in 1967.

POD is based on finding an optimal orthogonal basis for a given data set. The basis resides in the ascending arrangement of the eigenvalues. The projection on the basis first vector will have a bigger norm than the projection on the second vector and so on. This provides the most efficient method for extracting the large-scale structure dynamics from a complex, nonlinear flow. POD can be applied to experimental data obtained by advanced optical measurement techniques, such as Particle Image Velocimetry (PIV) or Planar Doppler Velocimetry (PDV). The POD analysis can also be performed on numerical data including Direct Numerical Simulation (DNS) data (Freund, Kastner et al.) and Large Eddy Simulation (LES) data.

The understanding of turbulent flows has been greatly advanced with the development of Computational Fluid Dynamics. Indeed this area has required an in-depth analysis and understanding of turbulence and turbulence models (Hinze, Kolmogorov, Spalart, Allmaras). These works lead eventually to a now well-recognized model: Large Eddy Simulation (LES). This model simulates the large-scale structures of the flow while modeling the smaller scales. It is a good compromise between the Reynolds-Average Navier Stokes (RANS) model, which is not able to capture turbulences structure and Direct Numerical Simulation (DNS), which requires a large amount of computational time for high Reynolds number flows.

In the present study POD has been performed on both numerical (LES) and experimental (PIV) data from an under-expanded axisymmetric jet. The PIV technique is non-intrusive, accurate, relatively quick to process, but only provides a planar measurement of the streamwise and radial velocity components. On the other hand LES has the advantage of providing data for the full domain as well as all flow variables: tangential velocity, density, pressure, and temperature. Traditionally, PIV does not provide any time information while LES does, however newer PIV systems are beginning to have time resolution well into the kilo-Hertz range. For the current application the laser power limits the PIV domain thus requiring stitching of sub-domains to create a larger picture of the flow. These restrictions do not exist with the LES data. In this study, the LES domain is broken up into sub-domains similar to the PIV data. A POD analysis is then performed to understand how measurements of the large-scale dynamics are influenced when the domain is broken down into these smaller sub-domains. The LES data set is also used to investigate the flow variables not available from the PIV database.

II. Experimental and Computational Arrangements

A. Particle Image Velocimetry

All Particle Image Velocimetry (PIV) experiments were performed in the Aeroacoustics Test Facility which is part of the Widen Tabakoff Gas Dynamics and Propulsion laboratory at the University of Cincinnati. The installation presents an axisymmetric nozzle, whose diameters at nozzle inflow, nozzle throat, and nozzle exit are 5.798, 2.640 and 2.868 inches, respectively. The area ratio is set for a design Mach number of 1.5. Data was taken for a variety of jet Mach numbers spanning the over-expanded, ideally-expanded, and under-expanded flow regimes. The LES data was only performed for a jet Mach number of 1.56 which is slightly under-expanded, so the main results are for this case which has a Normalized Pressure Ratio (NPR) of 4.0. The nozzle geometry is typical from a military engine nozzle. It presents a sharp junction between convergent and divergent sections.
PIV is a method that with a single camera can measure two velocity components, and with two cameras can measure all three velocity components by the stereo PIV technique \(^\text{ref}^3\). In the current study, only one camera was used to measure the streamwise (\(u\)) and radial (\(v\)) velocity components on a streamwise plane. The experimental arrangement features a seeded flow, a CCD camera and high pulse energy laser. The flow was seeded with Olive Oil using in-house built seeders which assured particle sizes on the order of a few microns to assure a good dynamical response to the flow. Details on seed particles size effects and limits can be found in Samimy and Werner\(^2\). More details on PIV method and equipment can be found in Adrian, Grant and Raffel et al. An ND:YAG laser was used for this study which provides short pulse duration (<10ns), pulse energies on the order of a 100mJ, and a harmonic generator to match the laser light color to the peak spectral response of the CCD camera\(^2\). The light sheet has a thickness on the order of 1 mm and is obtained by using spherical and cylindrical optics. The spatial resolution of PIV is limited by the ability to spread the laser pulse into a sheet and the spatial resolution of the camera. This limitation leads to breaking up the large domain into smaller sub-domains as seen in Figure 2. In the current jet studied at UC, seven sub-domains were used to span from the nozzle exit to 20 diameters downstream. Also, to account for the spread of the jet, it was necessary to shift the camera up from position 1 to 2 which subsequently cut off some of the bottom shear layer.

![Figure 1. Schematic of PIV setup at two different axial measurement positions.](image)

**B. Large Eddy Simulation**

Large Eddy Simulation (LES) has been performed on the same nozzle geometry used for experiments and at the slightly under-expanded case with an NPR of 4.0. LES method is based on the Kolomogorov’s theory (1941) postulating that the flow large-scale structures can be calculated and the small ones more universal can be modeled. The code used for the simulation is a finite-element Computational Fluid Dynamics (CFD) code (FEFLO), developed by Prof. Lohner and scientists in Naval Research Laboratory. The spatial discretization used for this simulation is the finite-element version of the Flux Corrected Transport (FEM-FCT). FCT was introduced by Boris and Book\(^17, 18, 19\) in the early 1970’s. This approach leads to a class of finite-difference algorithms which strictly enforce the nonnegative property of realistic mass and energy densities. Flux limiters are used to prevent the formation of spacious undershoots and overshoots in vicinity of steep gradients. As a result steep gradients and shocks can be handled particularly well. A fourth-order Taylor-Galerkin scheme is used for time discretization. No explicit subgrid-scale model is used and the embedded flux limiter implicitly provides the modeling of subgrid scales. The present LES is in the framework of Monotonically Integrated Large Eddy Simulations (MILES)\(^21\).

The computational domain spans 30 jet diameters (\(D\)) in the radial direction and 64 jet diameters in the streamwise direction. An unstructured tetrahedral grid is used to model the nozzle geometry. The grid contains a total of 11million grid points and 65 million tetrahedral elements. The flow is 3D unsteady, inviscid and compressible. The
nozzle boundary layer and inflow turbulence are not included in the simulation, so only turbulences generated by disturbances from the nozzle lip and instabilities growing in the jet are considered. More details regarding the simulation set-up can be found in Liu and Kailasanath (2009) 21.

C. Proper Orthogonal Decomposition
The Proper Orthogonal Decomposition (POD) is a mathematical method used to extract the most energetic structures from a turbulent flow. Lumley introduced it in 1967 as a means to better understand the dynamics of turbulent flows. The analysis principle is to solve an eigen-value problem. A spatial basis of eigen-functions is obtained which are termed the POD modes. Even if it is not proven, it is often accepted that the POD modes are connected to the coherent structures of the flow. Traditionally with experimental data, the vector field decomposed is the velocity field. The advantage of LES data is to enable the POD analysis on any flow variable, such as velocity, density, pressure or temperature.

The main word that should be remembered from the Proper Orthogonal Decomposition is “decomposition”. The ultimate goal is in fact to approximate a function \( f(x,t) \) as a finite sum in the variables separated form:

\[
f(x,t) = \sum_{i=1}^{M} a_i(t) \phi_i(x)
\]  

This equation has several representations. The decomposition could be a Fourier series, Chebyshev polynomials, Laplace series, or Legendre polynomials. The particular decomposition presented is defined such that the functions or vectors \( \phi_i \) are orthogonal to each other, their norm is equal to 1, and the approximation for each \( M \) is optimized in a least squares sense. In other words \( \phi_i \) is chosen such that the mean energy is maximized where the mean energy is defined as:

\[
a_i = \frac{\langle \langle \tilde{u}, \phi_i \rangle \rangle}{\langle \phi_i, \phi_i \rangle^{1/2}}
\]

with \( f(x,t) \) being replaced by the fluctuating velocity, \( \tilde{u} \).

This maximization leads to a Fredholm’s equation of the second kind homogeneous, which is an eigenvalue problem written below.

\[
\sum_{i=1}^{M} \int_{D} R_{ki}(x,x') \phi_i(x') dx' = \lambda \phi_k(x)
\]

where \( R_{ki}(x,x') \) is the cross-correlation tensor. This problem has a solution according to the Hilbert Schmidt theorem provided that the space used is a Hilbert space. Depending on the method used the cross-correlation tensor is defined differently. In the classical POD method introduced by Lumley \( R_{ki}(x,x') \) can be written:

\[
R_{ki}(x,x') = \frac{1}{T} \int_{T} u_k(x,t) u_i(x',t) dt
\]

In the present study the snapshot method is used which was introduced by Sirovich in 1987. It requires a large number \( N \) of instantaneous realizations or snapshots. The cross-correlation tensor here called the two point correlation tensor can be written as:

\[
R(t,t_k) = \frac{1}{N} \int_{D} u_i(x,t) u_i(x,t_k) dx
\]

The snapshot method is optimal when using PIV or LES data since both have more spatial points than time steps.

III. Results
Figure 2 presents the mean streamwise velocity for the under-expanded supersonic jet with a jet Mach number of 1.56. The results are consistent with an under-expanded jet in that the centerline velocity shows an initial acceleration of the flow at the nozzle exit due to the static pressure being higher than the ambient pressure. With downstream distance, the growth of the shear layer and weakening of the shock cell structures leads to a decay of the
jet velocity. The quality of PIV data was not very good past the X/D = 10. Three challenges were presented for the current PIV, at the higher Mach number (1) it was difficult to get enough seed into the flow, (2) there was a significant amount of condensation when the moist, warm ambient air was entrained into the cold jet, and (3) the increased spread of the jet lead to a diminishing quality of laser power as the entrained air scattered more laser light. However, the experiments were not just performed at this NPR instead a large range of NPR’s were investigated which will be presented later in the paper. It will be seen that the quality of data for the lower NPR’s was much better.

In a similar fashion as the PIV results, the mean streamwise velocity for the LES data is presented in Figure 3. A similar set of velocity features are seen for the LES results and have been compared in the past to the experimental results\(^1,2\). What is worth emphasizing due to its impact on POD is that the LES contains the entire domain and the data set is time correlated. When going from one measurement location to another with the PIV, it was necessary to stop the measurement, traverse the camera and laser, and then take data. Therefore there is no time correlation available in the PIV data. The LES data also includes the tangential velocity, pressure, density, and temperature. The POD modes will look into these different flow variables. Finally, only a streamwise view is shown to be consistent with the PIV, however, the computational domain includes the entire 3-D domain, which could allow an investigation into the azimuthal structure of the jet. This will be saved for future work.

![Figure 2. PIV measurement of mean axial velocity.](image)

![Figure 3. LES full domain mean axial velocity.](image)

Figure 4 presents the 1\(^{st}\) POD mode, $\phi_1(x)$, for the axial fluctuating velocity component on the left and the radial fluctuating velocity component on the right. The 1\(^{st}\) POD mode is presented because it extracts the highest energy flow features. Thus it makes up the largest percentage of TKE compared to any other POD mode. The colorbar scheme is from -1 (blue) to +1 (red) since the POD modes have been normalized. The top row corresponds to the PIV data, and the bottom three rows correspond to the LES data. Three different domain sizes are presented when performing the POD analysis on the LES data: the full domain, small domain, and PIV domain. The full domain corresponds to the region downstream of the nozzle exit until 25D and spans the radial dimension from -2D to + 2D. The full computational domain is larger but the full domain includes the region of high fluctuations which is essential for POD. After computing the POD modes for the full domain, the next step was to break down the full domain into smaller sub-domains to allow for a direct comparison to the PIV data. Thus, the width of the small domain was selected to match the streamwise span of the experimental PIV domain (2.6D). A total of eight small domains are used to span the full domain. After extracting data for the small domain, a POD analysis was done at each measurement station. The final step was to compress the small domain in the radial direction to have the exact dimensions and positions as the PIV domain. In this manner the comparison is even more precise.

For the full domain, large structures are observed between 9D and 24D for the axial velocity component and between 12D and 22D for the radial velocity component. This downstream region is where the centerline velocity begins to decay and the turbulence levels begin to peak near the jet centerline due to the collapse of the potential core. The flow structures between the nozzle exit and the end of the potential core have less energy and therefore are not part of the 1\(^{st}\) POD mode for the full domain. For both the axial and radial velocity components, the first
POD mode is representative of a helical mode. This is expected since after the potential core collapse, an axisymmetric jet is no longer unstable to the axisymmetric mode \((m = 0)\). The radial velocity is actually the transverse velocity where a blue feature in the top shear layer is entraining flow and a blue feature in the bottom shear layer is ejecting fluid away from the jet centerline. Similar sign issues between the top and bottom shear layers on a streamwise plane are seen when studying the \(\nu\) Reynolds stress component.

![Figure 4. First POD Mode from PIV and LES data. a) Axial Velocity, b) Radial Velocity.](image)

The small domain and PIV domain show the presence of organized structures between the nozzle exit and the end of the potential core. It is also seen in the downstream regions that the very large structures seen in the full domain have been filtered with the smaller domain size. This is particularly noticeable for the streamwise velocity. Three sub-domains span the region between 0D and 8D. At the two first positions, the coherent structures are located in the shear layer and are most prevalent at the downstream position of the sub-domain. They reside in the downstream region of the domain because the fluctuation levels grow downstream. For the 1st POD mode of the radial velocity component, the structures are either in the “top” or “bottom” shear layer upstream of 7D while after 7D the structures in the top and bottom shear layers begin to merge on the jet centerline. This is coupled with an increase in the jet centerline TKE and marking the collapse of the potential core. The streamwise velocity seems to lack this growth of intensity along the centerline, but these conclusions are only based on the 1st POD mode.

A comparison between the PIV data and the PIV domain for the LES shows very few similarities in the shape of the 1st POD mode. It appears that the PIV data has filtered out the first few POD modes, at least more so than the LES data. This is a qualitative comparison, so not much detail will be given yet to explain their differences. After comparing the energy distribution, a few direct comments will be given explaining some of the differences and how these differences can be resolved.

Figure 5 presents the 8th POD for the axial and radial velocities using both the PIV and LES databases. The features seen in higher POD modes tend to be smaller in wavelength and contain less energy by definition. There is still a noticeable difference when going from the full domain to the smaller domains for the LES data. However, the flow features are smaller in all domains and the spatial filtering is not as strong compared to the spatial filtering of the 1st POD mode. Also, when comparing the LES to PIV data there appears to be more similarities between the two databases for this higher mode. This provides further support that the PIV experiments filtered out the largest scales.
Figure 5. 8th POD Mode for PIV and LES data. a) Axial Velocity, b) Radial Velocity

Figure 6 presents the POD analysis for the three flow variables available from an LES database but not from a PIV database (tangential velocity, density, and pressure). The temperature POD modes are also available but not presented since this was a cold jet, and the thermal energy is low. The 1st POD mode for the tangential velocity has many similarities to the shape of the 1st POD mode for the radial velocity. Also, both velocity components had structures whose streamwise wavelengths were small enough that the spatial filtering when going to smaller domains was not as strong as the axial velocity component. The density and pressure features near the end of the potential core are very small compared to the velocity features and subsequently are the least influenced when going to smaller domains. In the full domain, both density and pressure present asymmetric mode pairs between 6D and 12D for the density and 10D and 15D for the pressure. The asymmetric mode having the highest energy was also seen for all three velocity components. An interesting result here is how the density structures are more upstream compared to all three velocity components and the pressure field. It appears the density features first break down, which leads to the quick growth and decay of a pressure feature just after X/D = 10. Finally, the pressure features break down around X/D = 12 which results in all three velocity components experiencing the growth, saturation and decay cycle of a large-scale asymmetric wave-packet.

Figure 6. 1st POD Mode on LES data for Tangential Velocity, Density, and Pressure.

Figure 7 presents the POD energy for the first 25 POD modes at the 7 measurement domains. The energy is presented for the axial and radial velocities obtained from PIV (top row) and the PIV domain for the LES data.
(bottom row). All figures have been normalized by the maximum modal amplitude of that dataset. The more tradition method of normalization at each measurement location will be presented shortly. The current presentation method allows there to be a comparison between locations in terms of absolute energy levels. As can be seen, there are many similarities in terms of energy distribution for the PIV and LES data. The energy in the first 10 to 15 modes is seen to grow until the 4th or 5th measurement domain for the axial velocity and the 6th domain for the radial velocity, after which the energy decays. The high energy for the PIV data axial velocity at Location 7 is due to the challenges with the measurement at that location. The LES data is seen to have the energy fall off to lower values quicker. This is probably due to the noise in the PIV data or because the LES is beginning to model some of these scales. It is known that a flatter distribution of energy is synonymous with a more random flow.

![Energy Distribution](image)

**Figure 7. Energy Distribution versus the number of Modes for a), b) Axial Velocity from PIV and LES respectively, and c), d) Radial Velocity from PIV and LES respectively.**

Figure 8 presents the energy contained in POD modes 1 to 4 at each location for the same data presented in Figure 7. The same trend is seen for radial velocity from LES and PIV. The most energetic position in both cases is the sixth. The axial velocities however show some differences between PIV and LES. Position 4 is a peak value in case of PIV, where in case of LES it is position 5. This can be explained by looking at Figures 2 & 3. It is seen that the potential core’s size on the numerical data is slightly over estimated compared to the experimental data. Since the present LES does not include the nozzle boundary layer and inflow turbulence, turbulence levels are much lower near the nozzle exit. This explains why the energy level predicted by LES is lower at the first one and two locations. The experimental data having another peak at location 7 is again attributed to data quality.
Figure 8. Energy Distribution in POD modes 1 to 4 versus measurement position

Figure 9. Energy Distribution in percent versus POD Mode number for the PIV domain
Figure 9 presents the energy distribution similar to Figure 7 but with each location normalized by its total energy. Presenting the data this way allows an investigation into the energy distribution instead of the energy levels. In general, the energy distribution collapses for the LES data. The first two radial POD modes do not obey this trend. The first measurement location seems to also have a flatter distribution of energy implying a more random flow near the nozzle exit. If the experimental data was plotted between 0 and 20 on the ordinate instead of 0 to 6 for the axial component and 0 to 3 for the radial component, the data would look more like the LES data. The percentage of energy in the first few modes is much higher for the LES relative to the experiments. It has been hypothesized that the PIV data has filtered out the largest structures in the POD analysis. This would explain why the energy levels are lower for the first few POD modes. Essentially if one were to shift the lines in Figure 9 for the PIV data to the right and have no data for the first few POD modes, then the energy distribution would be very similar between the two data sets. This is consistent with a comparison of the 1st POD modes shape in Figure 4. It is believed the relatively small window size in the PIV processing (16 x 16) is filtering the large-scale structures. Future work will look into processing the data with a larger window size of 32 x 32 and see if the data sets are a better match. This is preliminary data and is providing an excellent means into how to compare computational and experimental datasets using methods more advanced than just mean velocity and TKE profiles.

Figure 10 presents the energy distribution for all available flow variables from the LES dataset. The full domain is included in all the plots, and because the full domain is larger and more scales are included the distribution of total energy is more equally distributed between POD modes. The pressure has a collapse in energy distribution for the various locations similar to what seen for the axial velocity and radial velocity. The energy distribution is different for the tangential velocity and density at all measurement locations. This is particularly true for the first 5 POD modes with the tangential velocity and for the first 10 to 15 modes for the density. The tangential velocity is related to the spin of the large-scale structures and this would imply that only the largest features have a significant amount of spin. The density had a distinct change in the large-scale features for the small domain when going from measurement station 4 to 6 (see Figure 6), this results in a change of energy distribution. It appears that measurement locations 1 and 2 are grouped, 3 and 4 are grouped, and 5 and 6 are grouped. The grouping of locations 3 and 4 correspond to the region where features seen in the full domain were also seen in the small domain.

Figure 10. Energy Distribution versus POD Mode number for the LES data using the Small Domain

The relationship between the various flow variables can be studied using the POD modal amplification coefficient, \( a_i \) in Equation 1 and 2. The goal here is to see if the axial or radial velocity POD modes can be used to reconstruct
the pressure, density or tangential POD modes. This relationship could then be used with the PIV dataset where only two velocity components were measured. Linear Stochastic Estimation is used for this analysis since the relationship between POD modes is not necessarily one-to-one. In other words, to estimate the first POD mode of pressure may require more than just the first axial or first radial velocity POD mode. Stochastic Estimation provides the optimal transfer function from multiple input signals to multiple output signals. Spectral stochastic estimation would even be better for this optimization. However, PIV data does not provide time correlated images which makes it impossible to use such a method.

After generating the relationship between the modes, the success of the method is determined by comparing the estimated POD modal amplification coefficients to the actual POD modal amplification coefficient. Figure 11 presents this correlation using 50 POD modes from the streamwise fluctuating velocity (top) and from the radial fluctuating velocity (bottom) to estimate the first 50 POD modes for the other three flow variable. Domain locations 1, 4, and 7 along with the full domain are presented. A general trend for all cases is the correlation coefficient decreases with increasing POD mode number and/or increasing domain number. Also, the line plots have more oscillations with increasing mode number or location number. The streamwise velocity is best for estimating the amplitude of the pressure POD modes and the radial velocity is best for estimating the amplitude of the tangential velocity POD modes. This is supported by correlation coefficients near 1. Both components do an okay job at estimating the amplification of the density POD modes.

![Figure 11](image1.png)

**Figure 11.** Using 10 POD modes from the axial (top row) and radial (bottom row) velocity to estimate the first 10 POD modes for the tangential velocity, pressure and density.

Figure 12 presents the same results as in Figure 11 except now only 10 POD modes were used in forming the stochastic estimates. Compared to Figure 11, the pressure is still best predicted by the streamwise velocity and the tangential velocity is still best predicted by the radial velocity. However, across the board the correlation coefficients have dropped in amplitude. This shows that the higher POD modes for the streamwise and radial velocity are important when estimating another flow quantity.
Figure 12. Using 10 POD modes from the axial (top row) and radial (bottom row) velocity to estimate the first 10 POD modes for the tangential velocity, pressure and density.

Figure 13 presents the actual modal amplitude coefficient for the 1st and 25th POD mode for a set of 100 images from the full domain. This shows the temporal evolution of the coefficient for a given POD modes which is important for predicting the modal dynamics. The results are presented for the actual temporal evolution, and the temporal evolution predicted using the axial ($U$) or radial ($V$) velocity POD modes. These results are consistent with those in Figure 11 in that the radial velocity is best for predicting the tangential velocity and the axial velocity is best for predicting the pressure. The estimate is more accurate and less sensitive to the estimating variable for the 1st POD mode compared to the 25th POD mode. Even the behavior of the first density POD mode is well predicted.

The ability to estimate the amplitude of the POD modes for one flow variable using a separate variable is a valuable tool since PIV images only measure the streamwise and radial velocity components. This tool also provides the ability to show how similarities/differences between PIV experiments and LES in terms of axial or radial velocity lead to similarities/differences in other flow variables such as tangential velocity, pressure, or density. It should be remembered that this jet is not ideally expanded so the high correlations could be due to the presence of shock cells, future work will look into ideally expanded jets to determine the link between various flow variables. Also, we are not estimating the POD mode shapes here instead we are estimating the amplitude of the individual POD modes. Once we have a dataset where the PIV and LES have similar POD modes, it will be possible to use the LES POD modes and the estimated time coefficients from the PIV data to form a low-order flow reconstruction of the tangential velocity and pressure field, and albeit less accurate, an estimate of the density field.
Figure 13. Temporal evolution of the modal amplitude for the 1\textsuperscript{st} and 25\textsuperscript{th} POD mode.
The PIV database included results from a variety of nozzle pressure ratios (NPR). The mean axial velocity is shown in Figure 14 for a range of NPR’s from 2.5 to 5.0. The ideally-expanded jet would correspond to a NPR of 3.6. Each figure is normalized by the jet velocity obtained for that NPR if the flow were ideally-expanded. The dark red color corresponds to a value of 1.2 and blue values correspond to zero. The intensity of the shock cells decrease and the spacing increases when going from NPR 2.5 to 3.5. The shock cell spacing continues to increase for the NPR 4.0 to 5.0, however now these are the under-expanded cases. The potential core also increases as the NPR is increased. The challenge of measuring high-speed jet lies in the fact that condensation appears in the shear layer, which removes seeds from the flow. This leads to a decrease of data quality as seen for NPRs 3.5 to 4.5, particularly downstream of location 4. The measurements were initially designed to capture details of the shock cells, especially near the nozzle exit. This goal has been fully reached. The current analysis is just to perform a preliminary set of POD modes to help guide future work.

![Figure 14. Mean Velocities from PIV data at different jet Mach numbers (NPR)](image)

The first POD mode for both the axial and radial velocity is presented for the varying NPR’s in Figure 15. The 1\textsuperscript{st} axial velocity POD mode seems to abruptly change after the shock cells begin to disintegrate. It appears the shock cell spacing controls the large-scale structure wavelength and once the shock cell strength significantly diminishes, the structure size abruptly changes and moves to the outer region of the domain. The structure size at the downstream locations is now related to the spreading of the shear layer instead of the shock cell spacing. This abrupt change in structure size is usually associated with the merging or pairing of large-scale structures and could be an important feature of shock cell noise. There also appears to be some distinct differences between the over-expanded and under-expanded cases. First, the shock-cell spacing increases with increasing NPR and thus the most distinct features due to the shock cells move downstream and are more dispersed. Due to this increased shock cell spacing, the small sub-domains appear to filter more features at the high NPR cases. The lowest NPR has up to 4 shock cell patterns in location 1 while the highest NPR have only 1 shock cell pattern. The database could be normalized by the shock cell spacing instead of jet diameter. However, the sub-domain size is fixed with PIV the, and spatial filtering would still be a problem.
Figure 15. 1st POD mode from PIV data at different NPR: (left) Axial Velocity, (right) Radial Velocity

Figure 16 presents the percentage of energy in each mode for these different NPR’s at locations 1, 4 and 7. The plots confirm the two over-expanded cases (NPR 2.5 and 3.0) have a large amount of energy in the first two POD modes. These two modes are most likely related to the strong screech modes which are present in over-expanded jets. In location 4, the NPR 3.0 case contains 20% of energy in the first POD mode, when the other NPRs percentages don’t exceed 10%. The NPR 2.5 has already seen the shock cells break down by location 4 while the NPR 3.0 still has some strong dynamics between location 3 and 4 (see 1st radial POD mode in Figure 15). The ideally expanded case cannot be differentiated from the other cases and is most similar to the under-expanded cases (NPR 4.0 to 5.0). The nozzle design was based on the contours of military nozzle therefore it is impossible to obtain a shock free jet. After 10 POD modes for the streamwise velocity and 15 POD modes for the radial velocity, all curves merge.
IV. Conclusion

Proper Orthogonal Decomposition (POD) was performed on Large Eddy Simulation (LES) data and on Particle Image Velocimetry (PIV) data from an under-expanded axisymmetric jet. PIV measures the streamwise and radial velocity components along a streamwise plane while LES provides data for the full domain and includes all flow variables (three velocity components, density, pressure, and temperature). The PIV domain was limited by the laser power and required a series of sub-domains to capture the shear layer prior to and after the collapse of the potential core. A qualitative comparison between the POD modes from the LES and PIV data shows features similar in wavelength and shape particularly just downstream of the nozzle exit. An energy analysis shows that the biggest difference between the two cases was near the nozzle exit and believed to be the fact that nozzle boundary layer and inflow turbulence are not included in the simulation, which results a low turbulence level near the nozzle exit. At downstream positions, both cases have a similar distribution of POD modal energy. The analysis also shows that the large-scale flow features captured by the POD modes are strongly dependent on the domain size. Small domains are good for extracting the smaller-scale and low energy flow features near the nozzle exit. Further downstream the smaller domain spatially filters the largest flow features from the POD modes. The LES data is further post-processed to see if the streamwise and/or radial velocity can be used to estimate the tangential velocity, pressure, or density. It is found that the streamwise velocity can predict the pressure field, the radial velocity can predict the tangential velocity, and neither component does a good job at predicting the density. The prediction of flow variables is most likely due to the under-expanded nature of the jet and shows how PIV data can be used to further estimate other flow variables.

Future work could include a variety of paths. The first path could be to perform LES on an overexpanded jet and compare to the PIV data for the same flow conditions. Also, the processing of the PIV data will be explored to understand window sizing and filtering of data based on the window size. If a good match is seen between LES and PIV data, then the next effort would begin to try and project the PIV images on the LES POD modes. This would begin the analysis of trying to predict other flow variable from the PIV database.
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