Applications of High-Order Weighted Compact Nonlinear Scheme for Complex Transonic Flows

Xiaogang Deng¹, Guangxue Wang² and Guohua Tu³

State Key Laboratory of Aerodynamics, P.O. Box 211, Mianyang, 621000, P.R. China

and

Meiliang Mao⁴

China Aerodynamics Research and Development Center, P.O. Box 211, Mianyang, 621000, P.R. China

The results of the four drag prediction workshops (DPW) of AIAA indicate that the most current low-order CFD methods trend to give poor prediction for pitching moment of transonic aircraft configurations. The high-order weighted compact nonlinear scheme (WCNS) and algorithms for viscous terms as well as grid metrics are packaged together to establish a high-order code for complex geometries. The numerical results of the RAE2822 airfoil and the DLR-F6 wing-body configuration indicate that the accuracy for shock position and pitching moment may be improved by high-order methods.

I. Introduction

The American Institute of Aeronautics and Astronautics (AIAA) Applied Aerodynamics Committee have sponsored four drag prediction workshops (DPW) with the aim of assessing the state-of-the-art computational methods as practical aerodynamic tools for aerodynamic force and moment prediction of transonic aircraft configurations (http://aaac.larc.nasa.gov/tsab/cfdlarc/aiaa-dpw). Transonic flow occurs when there is mixed sub- and supersonic local flow in the same flowfield (typically with freestream Mach numbers from $\mathcal{M}_\infty = 0.6$ or 0.7 to 1.2). The transonic flow regime provides the most efficient aircraft cruise performance, hence, most large commercial aircraft cruise in this regime [20]. However, transonic flow fields tend to be sensitive to small perturbations in flow conditions or to slight changes in geometrical characteristics, either of which can cause large variations in the flow field [20]. This complicates both computations and wind tunnel testing. As noted by Hafez [21], transonic aerodynamics is a rich field full of challenging problems for CFD, for example, the discretization process introduces errors, without controlling these errors, the results can be misleading. The four DPW show that current CFD codes may produce successful aerodynamic prediction for the transonic configurations. However, the results are somewhat scattered, especially for shock location, separation bubble, and drag as well as pitching moment. As the numerical methods of the four DPW are mainly low-order ones (less than 3rd-order), the present invest [72] the performance of high-order WCNS for complex transonic flows.

It is said that high order algorithms are very useful in CFD due to the increased demand to accurately predict engineering problems and understand fundamental flow physics [1, 2]. It is generally believed that the accurate simulation of fluid flow with multiple and wide range of spatial scales and structures is a difficult task expect through spectral approximations. However, the use of spectral approximations is limited to simple geometries with generally periodic boundary conditions. Compact schemes make it possible to devise, on a given stencil, finite difference schemes that have much better resolution properties than conventional explicit finite difference schemes of comparable order of accuracy. Compact schemes with spectral-like resolution properties are more convenient to use than spectral and pseudo-spectral schemes, and are easier to handle, especially when nontrivial geometries are involved [3]. However, central algorithms are intrinsically non-dissipative, and can not prevent odd-even decoupling, which gives rise to high frequency oscillations even in smooth regions. Reducing or removing such oscillations
requires the introduction of dissipation terms. Upwind or upwind-based compact schemes with their dissipative properties are more stable than central ones. Deng [4] have proposed a type of one-parameter linear dissipative compact schemes (DCS), which was derived as to damp out the dispersive and parasite errors in the high-wave-number regions. Filters are also can be applied to prevent numerical oscillations, such as the ones proposed by Visbal [5, 6]. Nevertheless, in the transonic and supersonic flow regions when dealing with flows involving shock waves, one must use a numerical scheme which can both represent small scale structures with the minimum of numerical dissipation and capture discontinuities with the robustness that is common to Godunov-type methods. To achieve these dual objectives, Deng [7, 8] have developed a series of weighted compact nonlinear schemes (WCNS). The WCNS-E-5, a typical explicit one of the weighted compact nonlinear schemes, has been successfully applied to a wide range of flow simulations so far to show its flexibility, robustness, and freestream and vortex preservation properties by Liu [9], Deng [10] and Nonomura [11].

In most published articles or reviews, the flows are complex with shocks, vortices, and turbulent structures, while the grids are relatively simple. The state-of-the-art applications of high-order and high resolution methods for engineering practices where the grids are complex are still limited. Although low-order schemes (second-order schemes) are widely used for engineering applications, they are insufficient for many viscosity dominant phenomena, such as boundary layer flows, vortical flows, separation flows, shock-boundary layer interactions, heat flux transfers, Reynolds-number effect simulations, etc. An effective approach to overcome the obstacles of accurate numerical simulations is to employ high-order and high resolution schemes which have been highlighted by Fujii when he summarize the latest progresses of computational fluid dynamics (CFD) as well as pointed out the three trends for future CFD [12, 13]. Over the past 20 to 30 years, there have been a lot of studies in developing and applying high-order numerical methods for CFD. However, challenges still remain in the implementations of high-order schemes for complex configurations where the grid quality is usually low. A series studies have been done by Deng et al. [10, 14, 17, 24] to implement high-order WCNS for complex configurations.

The engineering prospects about the applications of the WCNS-E-5 for complex transonic configurations are discussed in this paper. The RAE2822 airfoil and the DLR-F6 wing-body configuration which was test in the DPW-II, are chosen in this paper to evaluate the high-order WCNS strategy for transonic flows. As shock waves and shock positions will lead to a rapid change in pitching moment, more attentions are focused on the shocks.

II. Governing Equations and Turbulence Model

A. Governing Equations

The Reynolds-Averaged Navier-Stokes (RANS) equations are chosen as governing equations. In Cartesian coordinates ($x$, $y$, $z$), the three-dimensional nondimensional strong conservative RANS equations are

$$
\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial G}{\partial z} = \left( \frac{\partial E_v}{\partial x} + \frac{\partial F_v}{\partial y} + \frac{\partial G_v}{\partial z} \right),
$$

where $Q = [\tilde{\rho}, \tilde{\rho}u, \tilde{\rho}v, \tilde{\rho}w, \tilde{\rho}e]^T$, $E$, $F$, $G$ and $E_v$, $F_v$, $G_v$ are the inviscid and viscous fluxes respectively, which can be found in many CFD textbooks. The governing equations (1) can be transformed in curvilinear coordinates by introducing the transformation $(x, y, z) \rightarrow (\xi, \eta, \zeta, \tau)$

$$
\frac{\partial \tilde{Q}}{\partial \tau} + \frac{\partial (\tilde{E} - s \cdot \tilde{E}_v)}{\partial \xi} + \frac{\partial (\tilde{F} - s \cdot \tilde{F}_v)}{\partial \eta} + \frac{\partial (\tilde{G} - s \cdot \tilde{G}_v)}{\partial \zeta} = 0.
$$

Here, $\tilde{Q} = Q / J$, and $J = |\partial(\xi, \eta, \zeta) / \partial(x, y, z)|$ is the Jacobian of coordinate transformation. The fluxes in the curvilinear coordinates are

$$
\tilde{E} = \xi_x Q + \xi_y E + \xi_z G, \quad \tilde{F} = \eta_x Q + \eta_y E + \eta_z F + \eta_z G, \quad \tilde{G} = \zeta_x Q + \zeta_y E + \zeta_z F + \zeta_z G,
$$

$$
\tilde{E}_v = \xi_x E_v + \xi_y F_v + \xi_z G_v, \quad \tilde{F}_v = \eta_x E_v + \eta_y F_v + \eta_z G_v, \quad \tilde{G}_v = \zeta_x E_v + \zeta_y F_v + \zeta_z G_v,
$$

where $\xi_x = J^{-1} \xi_x$ and with similar definition to other metrics.
B. Turbulence Model

The Spalart-Allmaras one-equation model [23] without the ‘trip’ function of \( f_{11} \) and \( f_{12} \) are chosen here. The nondimensional model equation can be written as

\[
\frac{D\rho\hat{v}}{Dt} = C_{b1}\rho\hat{S}\hat{v} + \frac{\rho}{Re\sigma} \left\{ \nabla \cdot \left[ (\nu + \nu^* \nabla \cdot \hat{v}) \right] + C_{b2} (\nabla \cdot \hat{v})^2 \right\} - \frac{\rho}{Re} C_{w1} f \left( \frac{\hat{v}}{d} \right)^2, \quad \nu_i = \nu f v_i
\]

and the coefficients in (5) can be found in [23].

III. Numerical Methods

A. Numerical Methods for Inviscid Terms

The explicit 5th-order weighted compact nonlinear scheme (WCNS-E-5) developed by Deng [7, 8] is adopted for inviscid terms, for example, \( \frac{\partial E}{\partial z} \) can be acquired by the following procedures

\[
\frac{\partial \hat{E}}{\partial \xi} = \frac{75}{64\Delta \xi} (\hat{E}_{i+1/2,j,k} - \hat{E}_{i-1/2,j,k}) - \frac{25}{384\Delta \xi} (\hat{E}_{i+3/2,j,k} - \hat{E}_{i-3/2,j,k}) + \frac{3}{640\Delta \xi} (\hat{E}_{i+5/2,j,k} - \hat{E}_{i-5/2,j,k}),
\]

where the flux at cell-edges is

\[
\hat{E}_{i+1/2,j,k} = \hat{E}^*(Q_{Li+1/2,j,k} + Q_{Ri+1/2,j,k} + \tilde{E}_{xi+1/2,j,k} + \tilde{E}_{xi+1/2,j,k}),
\]

and the cell edge variables \( Q_{Li+1/2,j,k} \) and \( Q_{Ri+1/2,j,k} \) are fifth-order weighted interpolations and have been derived by Deng and Zhang [7]. \( \hat{E}^* \) are the flux splittings which can also be found in [7]. The cell-edge metrics can be obtained by the 4th-order Lagrange interpolation

\[
\tilde{E}_{xi+1/2,j,k} = \frac{1}{16} (\tilde{E}_{xi-1,j,k} + 9\tilde{E}_{xi,j,k} + 9\tilde{E}_{xi+1,j,k} - \tilde{E}_{xi+2,j,k}).
\]

B. Numerical Methods for Viscous Terms

The numerical methods for viscous terms shall also be high-order ones. However, the direct extend of low-order central schemes to high-order central schemes is not a good choice as numerical oscillations may appear around singular points as well as sharp gradient regions. In order to enhance the numerical stability, a kind of nonlinear scheme for viscous terms is prepared by Liu [24] in another article. Liu’s methods are applied in this paper.

C. Numerical Methods for Grid metrics

The numerical methods for grid metrics are derived for the ‘conservative metric method (CMM)’ which can ensure the surface conservation law (SCL) for high-order difference schemes including boundaries. For example, \( \xi = J^{-1} \xi_x \) is calculated by

\[
\xi_x = \left( y_\xi \right)_\zeta - \left( y_\zeta \right)_\xi
\]

and the derivatives in (8) are acquired by

\[
\frac{\partial a_i}{\partial \xi} = \frac{75}{64\Delta \xi} \left[ a_{i+1/2} - a_{i-1/2} \right] - \frac{25}{384\Delta \xi} \left[ a_{i+3/2} - a_{i-3/2} \right] + \frac{3}{640\Delta \xi} \left[ a_{i+5/2} - a_{i-5/2} \right]
\]

where
\[ a_{i+1/2} = \frac{1}{16} \left( -a_{i-1} + 9a_i + 9a_{i+1} - a_{i+2} \right) \]  \hspace{1cm} (11)

D. Multi-Block Treatments

Generally speaking, it is difficult to generate a high-quality structured single-block grid system for a complex configuration. Point-matched multi-block structured grids are generated for complex configurations.

IV. Results and Discussion

A. RAE2822 Airfoil

The profile, RAE2822, for which the simulations were carried out, has been extensively used for validation of Navier-Stokes codes applied to transonic airfoil flow. There is experimental work available which includes boundary layer profiles, pressure distributions, displacement and momentum thicknesses [18]. There are also a large number of simulations carried out with different flow parameters and different turbulence models. In this work, the “case 9” denoted by in Cook [18] and Hellström [19] is selected to validate the code. The flow is supposed to be full turbulence in our simulations. The parameters are described below

\[ M_\infty = 0.73, \quad \alpha = 2.79^\circ, \quad Re = 6.5 \times 10^6 \]

Table 1 provides a description of the grids used and Fig 1 shows the medium grid. All of the grids have a “C” topology, and it extends 25 chord lengths in front, upper and lower side as well as downstream of the trailing edge. The coarse, medium, and fine grid have roughly 10845, 23985, and 53253 nodes, respectively. For all grids and the case, the \( y^+ \) is the standard law-of-the-wall coordinate, and therefore there are a few grid points in the linear sublayer of the turbulent boundary layers.

Except the high-order methods mentioned above, the second-order method is applied in the present study for comparison simulations. Here ‘low-order’ mainly refers to second-order method.

Fig.2 shows the grid convergence of the lift coefficient, \( C_n \), and drag coefficient, \( C_d \), for the test case computed. The results are plotted vs \( N^{-5/2} \) where \( N \) is the total number of grid nodes. Results are shown for the present higher-order(WCNS-E-5), and it suggest that the solutions obtained on the fine grid independent, difference between the medium and fine grid is very small. Therefore, the fine grid is used in the following.

Fig.3 shows velocity profiles from the upper surface boundary layer at stations of \( x/c = 0.179, 0.319, 0.404, 0.498, 0.574, 0.650, 0.750, 0.900 \) using the fine grid. The present methods shows better agreement with experimental velocity profiles in the upstream region before the shock wave.

Fig.4 shows the Friction coefficients and The wall pressure coefficients are shown in Fig. 5, which indicate that the accuracy of shock position is improved by the high-order WCNS. The lift coefficients (CL), drag coefficients (CD), and pitching moment (CMz) are listed in Table 2. It can be found that good consistencies are acquired between computed results and experimental results, and the accuracy of CL and CMz is improved by applying the high-order WCNS. We conjecture that the accuracy of CL and CMz is related to that of the shock position.
Fig. 2 Grid convergence study for Case 9
Fig. 3 Velocity profiles on the upper surface of the RAE2822 airfoil

Fig. 4 Friction coefficients

Fig. 5 Surface pressure coefficients
Table I  Grids of the RAE2822

<table>
<thead>
<tr>
<th>Grid</th>
<th>Total Nodes</th>
<th>Dimensions</th>
<th>Points on Airfoil</th>
<th>Points on Wake Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>10845</td>
<td>241*45</td>
<td>201</td>
<td>20</td>
</tr>
<tr>
<td>Medium</td>
<td>23985</td>
<td>369*65</td>
<td>305</td>
<td>33</td>
</tr>
<tr>
<td>Fine</td>
<td>53253</td>
<td>549*97</td>
<td>453</td>
<td>49</td>
</tr>
</tbody>
</table>

Table II  Results of the RAE2822

<table>
<thead>
<tr>
<th></th>
<th>CL</th>
<th>△CL(%)</th>
<th>CD</th>
<th>△CD(%)</th>
<th>CMz</th>
<th>△CMz(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXP</td>
<td>0.803</td>
<td></td>
<td>0.0168</td>
<td></td>
<td>-0.099</td>
<td></td>
</tr>
<tr>
<td>WCNS</td>
<td>0.8019</td>
<td>-0.1%</td>
<td>0.0177</td>
<td>5.4%</td>
<td>-0.0926</td>
<td>-6.5%</td>
</tr>
<tr>
<td>Second-order</td>
<td>0.7729</td>
<td>-3.7%</td>
<td>0.0160</td>
<td>-4.8%</td>
<td>-0.0861</td>
<td>-13%</td>
</tr>
</tbody>
</table>

B. DLR-F6 Wing-Body Configuration

The present study is conducted according to the DPW-II. Multi-block point-matched structured grids are used. The grids are O-type topology around the fuselage and C-H topology around the wing. The near wall spacing for viscous resolution of the wing is about 0.001 mm. The grids are shown in Fig. 6. The flow conditions and reference quantities are listed in Table 4.

Except the high-order methods mentioned above, the second-order method applied in the present study for comparison simulations is the same method mentioned in section A.

The results of the 4.287-million grids are shown in Fig. 7 and Fig. 8. Fig. 7 indicates that accuracy for pitching moment may be noticeably improved by the high-order WCNS. The lift coefficients acquired by the WCNS and the Second-order method agree well with each other, and both are consistent with the experiment results.

It should be noted that the Second-order method can be regarded as a congenor of the various numerical methods adopted in the DPW-II. When comparing our results to that of the DPW-II [25], one can find that the accuracy of the Second-order method is comparable to that of the most methods in the DPW-II. It can be found that the WCNS gives better results for pitching moment and lift coefficients than that of many methods adopted in the DPW-II.

In Fig. 9, the Cp distributions at four span locations are given. It can be found that the results are improved by the WCNS. The four maps indicate that the WCNS is superior than the Second-order method in the prediction of shock positions, as has been shown in the RAE2822 airfoil case. We suspect that the shock positions as well as the pressure distributions are caused to improve the accuracy of pitching moment.

Figure 10 shows the high-order methods predicted pressure surface contours and streamlines in the area of the trailing edge wing-body juncture separation bubble.

Table III  Flow conditions and reference quantities of DLR-F6 wing-body

<table>
<thead>
<tr>
<th>Flow conditions</th>
<th>Ma = 0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Re = 3 x10⁶</td>
</tr>
</tbody>
</table>

Half model reference area  | S/2 = 72700mm² |
Mean aerodynamic chord     | Cref = 141.22mm |
Projected half span        | b/2 = 585.647mm |
Aspect ratio               | AR = 9:5     |

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Fig. 6  Grids of the DLF-F6.

(a) Surface mesh on wing, body and symmetry plane.

(b) Slice mesh near the wing trailing edge.

Fig. 7  Pitching moment.

Wing/Body

\[ \text{M} = 0.75 \]

\[ \text{Re} = 3.0 \times 10^5 \]

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Fig. 8 Lift curves.

Fig. 9 $C_p$ distributions at 15%, 33.1%, 41.1% and 51.4% span locations.
V. Conclusion

The explicit 5th-order weighted compact nonlinear scheme (WCNS-E-5) can be implemented as a practical aerodynamic tool for force and pitching moment prediction of industry relevant geometries. The shock position as well as the pitching moment of the transonic RAE2822 airfoil and the DLR-F6 may be improved by the high-order method.

Acknowledgments

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References


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